Name: _____

Precalculus

Trigonometric Functions

I. AMC (Merrill) Practice Worksheets
II. FMC (Connally) Exercises & Problems
III. PTCA (Foerster) Explorations
IV. } N Other Worksheets
V. } Formulas and notes

v_2.0 2007-2008

5-1

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Practice Worksheet

Angles and Their Measure

If each angle has the given measure and is in standard position, determine the quadrant in which its terminal side lies.

1. $\frac{7\pi}{12}$ **2.** $-\frac{2\pi}{3}$ **3.** 371° **4.** $\frac{14\pi}{5}$

5. −156° **6.** 1000° **7.** 332° **8.** −240°

Change each degree measure to radian measure in terms of π.

9. 36° **10.** −250° **11.** −145° **12.** 6°

13. 870° **14.** 18° **15.** −820° **16.** 345°

Change each radian measure to degree measure.

17. −1 **18.** 4π **19.** −2.56 **20.** 12.85

21. $\frac{3\pi}{16}$ **22.** $-\frac{7\pi}{9}$ **23.** $\frac{13\pi}{30}$ **24.** $-\frac{17\pi}{3}$

Find one positive angle and one negative angle that are coterminal with each angle.

25. 70° **26.** $-\frac{2\pi}{5}$ **27.** −300° **28.** $\frac{3\pi}{4}$

Find the reference angle for each angle with the given measure.

29. −20° **30.** 160° **31.** −545° **32.** 300°

33. $\frac{10\pi}{3}$ **34.** $-\frac{5\pi}{8}$ **35.** $-\frac{\pi}{4}$ **36.** $-\frac{7\pi}{3}$

5-2

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Practice Worksheet

Central Angles and Arcs

Given the radian measure of a central angle, find the measure of its intercepted arc in terms of π in a circle of radius 10 cm.

1. $\frac{\pi}{6}$ **2.** $\frac{\pi}{3}$ **3.** $\frac{\pi}{2}$ **4.** $\frac{\pi}{5}$

5. $\frac{3\pi}{5}$ **6.** $\frac{4\pi}{7}$ **7.** $\frac{\pi}{12}$ **8.** $\frac{\pi}{24}$

Given the measurement of a central angle, find the measure of its intercepted arc in terms of π in a circle of diameter 60 in.

9. $10°$ **10.** $60°$ **11.** $42°$ **12.** $50°$

13. $72°$ **14.** $110°$ **15.** $35°$ **16.** $65°$

Given the measure of an arc, find the degree measure to the nearest tenth of the central angle it subtends in a circle of radius 16 cm.

17. 87 cm **18.** 5.6 cm **19.** 12 cm **20.** 25 cm

21. 10.24 cm **22.** 7.9 cm **23.** 11 cm **24.** 6 cm

Find the area of each sector to the nearest tenth, given its central angle, θ, and the radius of the circle.

25. $\theta = \frac{\pi}{6}$, $r = 14$ cm **26.** $\theta = \frac{\pi}{6}$, $r = 12$ ft

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5-3

Practice Worksheet

Circular Functions

Find the values of the six trigonometric functions of an angle in standard position if the given point lies on its terminal side.

1. $(-1, 5)$ **2.** $(6, -8)$ **3.** $(3, 2)$ **4.** $(-3, -4)$

5. $(0, -4)$ **6.** $(7, 0)$ **7.** $(\sqrt{2}, -\sqrt{2})$ **8.** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Suppose θ is an angle in standard position whose terminal side lies in the given quadrant. For each function, find the values of the remaining five trigonometric functions of θ.

9. $\cos \theta = \frac{3}{5}$; quadrant I

10. $\sin \theta = -\frac{2}{3}$; quadrant IV

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5-4

Practice Worksheet

Trigonometric Functions of Special Angles

Find each exact value. Do not use a calculator.

1. $\sin \frac{\pi}{4}$ **2.** $\cos \frac{\pi}{4}$ **3.** $\tan \frac{\pi}{4}$

4. $\cos 210°$ **5.** $\sin 300°$ **6.** $\tan 330°$

7. $\sin \frac{3\pi}{4}$ **8.** $\cos \frac{3\pi}{4}$ **9.** $\tan \frac{3\pi}{4}$

10. $\sin 90°$ **11.** $\csc 270°$ **12.** $\tan 45°$

13. $\cos \frac{3\pi}{2}$ **14.** $\tan \frac{3\pi}{2}$ **15.** $\sin \frac{3\pi}{2}$

Use a calculator to approximate each value to four decimal places.

16. $\cot (-75°)$ **17.** $\sin 634°$ **18.** $\cos 235°$

19. $\sin 2$ **20.** $\sec 4.28$ **21.** $\cot 0.23$

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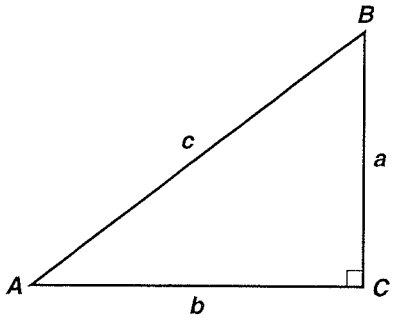
5-5

Practice Worksheet

Right Triangles

Solve each triangle described, given the triangle below. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. $A = 39°12', b = 2.1$



2. $a = 9, B = 49°$

3. $B = 64°, b = 19.2$

4. $B = 56°48', c = 63.1$

5. $A = 16°, c = 14$

6. $a = 0.4, c = 0.5$

7. $c = 21.3, A = 26°20'$

8. $a = 2, b = 7$

9. $A = 55°55', c = 16$

10. $a = \sqrt{15}, B = 18°$

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6-1

Practice Worksheet

Graphs of the Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1. $\sin(-720°)$

2. $\tan(-180°)$

3. $\cos(540°)$

4. $\tan(180°)$

5. $\csc(720°)$

6. $\sec(180°)$

Find the values of θ for which each equation is true.

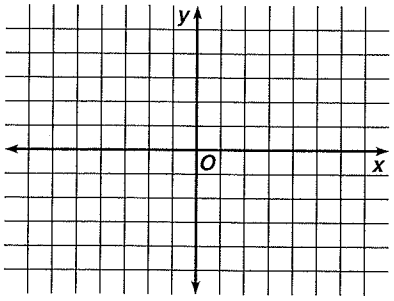
7. $\sin\theta = -1$

8. $\sec\theta = -1$

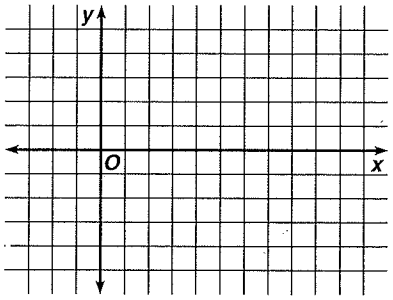
9. $\tan\theta = 0$

Graph each function on the given interval.

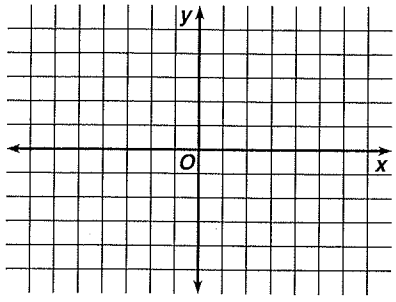
10. $y = \sin x; -90° \leq x \leq 90°$



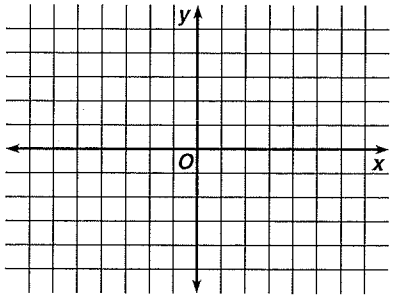
11. $y = \tan x; -90° \leq x \leq 270°$



12. $y = \cos x; -360° \leq x \leq 360°$



13. $y = \sec x; -360° \leq x \leq 360°$



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6-2 Practice Worksheet

Amplitude, Period, and Phase Shift and Horizontal Shift.

State the amplitude, period, and phase shift and horizontal shift

1. $y = -2 \sin \theta$
2. $y = 10 \sec \theta$
3. $y = -3 \sin 4\theta$
4. $y = 0.5 \sin\left(\theta - \frac{\pi}{3}\right)$
5. $y = 2.5 \cos(\theta + 180°)$
6. $y = -1.5 \sin\left(4\theta - \frac{\pi}{4}\right)$

Write an equation of the sine function with each amplitude, period, and horizontal ^ ***shift.***

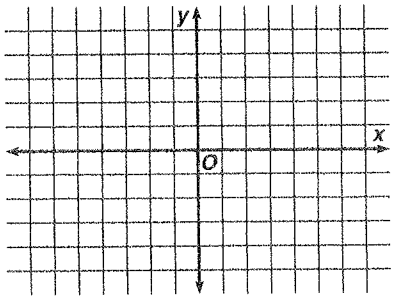
7. amplitude = 0.75, period = 360°, horizontal ^ shift = 30°
8. amplitude = 4, period = 3°, horizontal ^ shift = –30°

Write an equation of the cosine function with each amplitude, period, and phase shift.

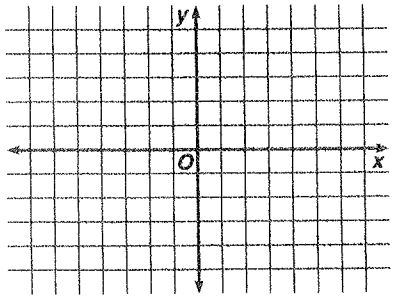
9. amplitude = 3.75, period = 90°, phase shift = 4°
10. amplitude = 12, period = 45°, phase shift = 180°

Graph each function.

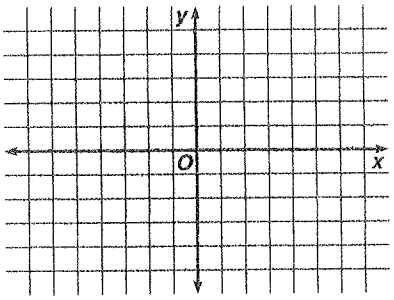
11. $y = 0.5 \sin x$



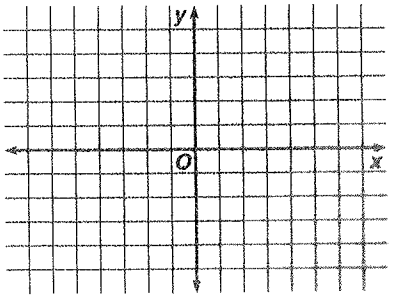
12. $y = 2 \cos(3x)$



13. $y = 2 \cos(2x - 45°)$



14. $y = \tan(x + 60°)$



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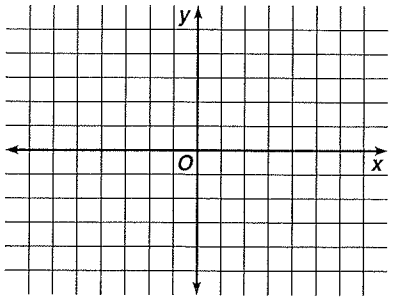
6-3

Practice Worksheet

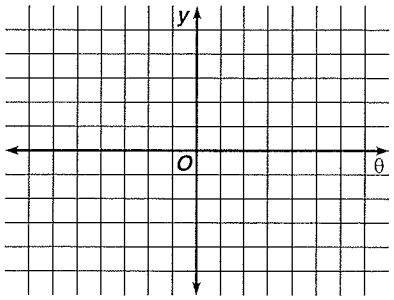
Graphing Trigonometric Functions

Graph each function.

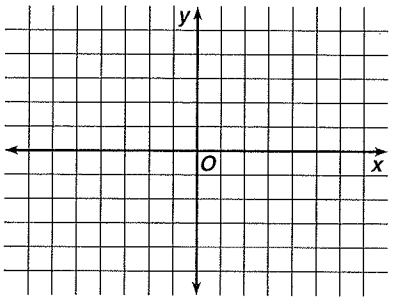
1. $y = 2 \sin (x - 45°)$



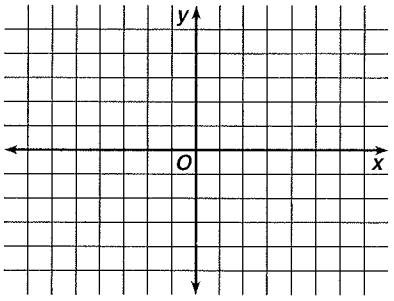
2. $y = -2 \cos (3\theta)$



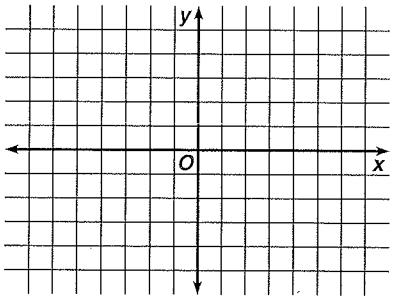
3. $y = \frac{1}{2} \cos \left(x - \frac{\pi}{2}\right)$



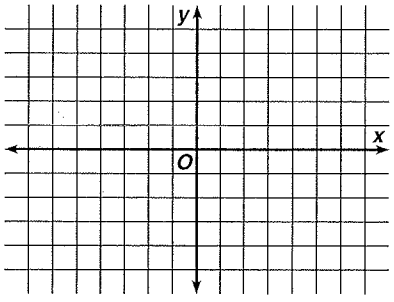
4. $y = \sin \left(\frac{x}{2} + 90°\right)$



5. $y = \sin x + \cos x$



6. $y = \cos 2x - \cos x$



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6-4

Practice Worksheet

Inverse Trigonometric Functions

Write each equation in the form of an inverse relation.

1. $0.75 = \sin x$

2. $-1 = \cos x$

3. $0.1 = \tan \theta$

4. $\frac{3}{5} = \cos x$

5. $\sin x = \frac{\sqrt{3}}{2}$

6. $\cos \alpha = \frac{12}{13}$

Find the values of x in the interval $0° \leq x \leq 360°$ that satisfy each equation.

7. $x = \arccos 1$

8. $\arccos \frac{\sqrt{2}}{2} = x$

9. $\arcsin \frac{1}{2} = x$

10. $\sin^{-1}(-1) = x$

11. $\sin^{-1} \frac{\sqrt{2}}{2} = x$

12. $\cot^{-1} 1 = x$

Evaluate each expression. Assume that all angles are in Quadrant I.

13. $\cos\left(\cos^{-1} \frac{1}{2}\right)$

14. $\sin\left(\cos^{-1} \frac{1}{2}\right)$

15. $\cos\left(\sin^{-1} \frac{1}{2}\right)$

16. $\tan\left(\sin^{-1} \frac{\sqrt{2}}{2} - \cos^{-1} \frac{\sqrt{2}}{2}\right)$

17. Verify that $\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = 90°$. Assume that all angles are in Quadrant I.

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6-5

Practice Worksheet

Principal Values of the Inverse Trigonometric Functions

Find each value.

1. Arcsin (–1)

2. Arccos 1

3. Arctan (–1)

4. $\text{Cos}^{-1}\ \frac{1}{2}$

5. Arcsin 1

6. $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

7. $\cos\left(\text{Cos}^{-1}\left(-\frac{1}{2}\right)\right)$

8. $\sin\left(\text{Sin}^{-1}\ \frac{\sqrt{3}}{2}\right)$

9. $\tan\left(\text{Tan}^{-1}\ \frac{\sqrt{3}}{3}\right)$

10. $\text{Cos}^{-1}\left(\text{Cos}\ \frac{\pi}{2}\right)$

11. $\text{Sin}^{-1}\left(\sin\ \frac{\pi}{4}\right)$

12. $\text{Tan}^{-1}\left(\tan\ \frac{\pi}{3}\right)$

13. $\cos\left(\text{Arcsin}\ \frac{1}{2}\right)$

14. $\sin\left(\text{Arccos}\frac{\sqrt{3}}{2}\right)$

15. $\tan\left(\text{Arcsin}\ \frac{\sqrt{3}}{3}\right)$

16. $\tan\left(\frac{1}{2}\ \text{Arccos}\ \frac{5}{13}\right)$

17. $\cos\left(\frac{1}{2}\ \text{Arcsin}\ \frac{6}{10}\right)$

18. $\sin\left(2\ \text{Arccos}\ \frac{3}{5}\right)$

19. $\sin\left[\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right)-\frac{\pi}{4}\right]$

20. $\cos\left[\text{Sin}^{-1}\left(\frac{\sqrt{2}}{2}\right)+\frac{\pi}{4}\right]$

21. $\text{Tan}\left[\frac{3\pi}{4}+\text{Sin}^{-1}\ \frac{\sqrt{2}}{2}\right]$

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6-6

Practice Worksheet

Graphing Inverses of Trigonometric Functions

State the domain and range of each relation.

1. $y = \text{Sin } x + 1$

2. $y = \sin x + 1$

3. $y = \cos x - 1$

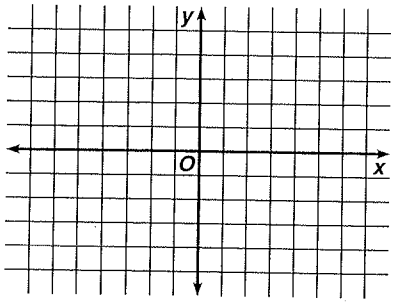
4. $y = \text{Cos}^{-1} x$

5. $y = \arcsin x$

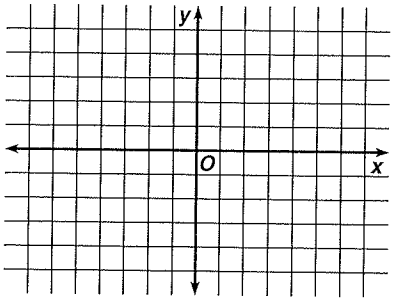
6. $y = \text{Tan}^{-1} x$

Write the equation for the inverse of each function. Then graph the function and its inverse.

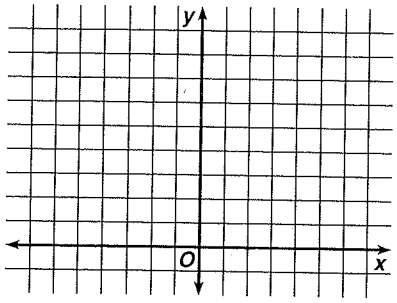
7. $y = \text{Cos}^{-1} x$



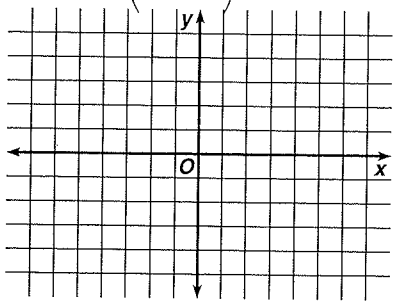
8. $y = \text{Tan}^{-1} (3x)$



9. $y = \frac{\pi}{2} + \text{Cos}^{-1} x$



10. $y = \text{Sin}\left(x - \frac{\pi}{2}\right)$



Determine if each of the following is true or false. If false, give a counterexample.

11. $\text{Cos}^{-1} x = \text{Cos}^{-1} (-x)$

12. $\text{Sin}^{-1} x = -\text{Sin}^{-1} x$

7-1

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Practice Worksheet

Basic Trigonometric Identities

Use Pythagorean Identities (not triangles)

Solve for values of θ between 0° and 90°.

1. If $\tan \theta = 2$, find $\cot \theta$.

2. If $\sin \theta = \frac{2}{3}$, find $\cos \theta$.

3. If $\cos \theta = \frac{1}{4}$, find $\tan \theta$

4. If $\tan \theta = 3$, find $\sec \theta$.

5. If $\sin \theta = \frac{7}{10}$, find $\cot \theta$.

6. If $\tan \theta = \frac{7}{2}$, find $\sin \theta$.

Express each value as a function of an angle in Quadrant I.

7. $\sin 458°$

8. $\cos 892°$

9. $\tan (-876°)$

10. $\csc 495°$

Simplify.

11. $\dfrac{\cot A}{\tan A}$

12. $\dfrac{\sin^2 \beta \cot \beta}{\cos \beta}$

13. $\sin^2 \theta \cos^2 \theta - \cos^2 \theta$

14. $\cos x + \sin x \tan x$

15. $\dfrac{1}{\sin^2 \theta} - \dfrac{\cos^2 \theta}{\sin^2 \theta}$

16. $\dfrac{\cos^2 \theta}{1 + \sin \theta}$

*17. $\dfrac{\tan^2 \theta - 1}{\tan^2 \theta + 1} + 2\cos^2 \theta$

*18. $1 - \dfrac{\sin^2 \theta}{1 + \cos \theta}$

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7-2

Practice Worksheet

Verifying Trigonometric Identities

Verify that each of the following is an identity.

1. $\frac{\csc x}{\cot x + \tan x} = \cos x$

2. $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

3. $\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2\sec^2 y$

4. $1 - 2\sin^2 r + \sin^4 r = \cos^4 r$

5. $\tan u + \frac{\cos u}{1 + \sin u} = \sec u$

6. $\frac{\tan x + \sec x}{\sec x - \cos x + \tan x} = \csc x$

Find a numerical value of one trigonometric function of each x.

7. $\sin x = 3 \cos x$

8. $\cos x = \cot x$

FMC 6.1 A

Exercises and Problems for Section 6.1

Exercises

In Exercises 1–8, do the functions appear to be periodic with period less than 4?

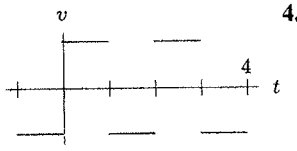
1.

t	0	1	2	3	4	5	6
$f(t)$	1	5	7	1	5	7	1

2.

r	0	π	2π	3π	4π	5π	6π	7π
$q(r)$	0	1	0	-1	1	0	1	0

3.



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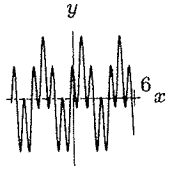

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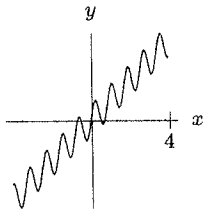

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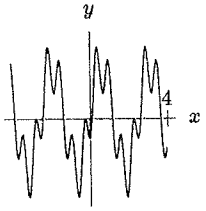

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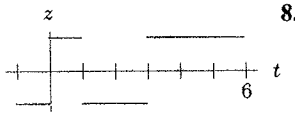

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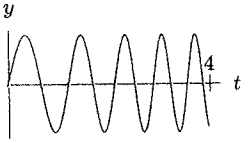


In Exercises 9–12, estimate the period of the periodic functions.

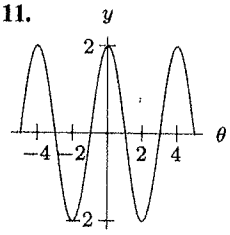
9.

t	0	1	2	3	4	5	6
$f(t)$	12	13	14	12	13	14	12

10.

z	1	11	21	31	41	51	61	71	81
$g(z)$	5	3	2	3	5	3	2	3	5

11.



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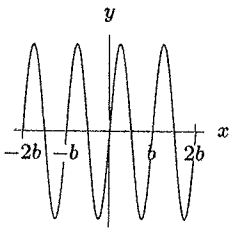


30. Table 6.3 gives the height $h = f(t)$ in feet of a weight on a spring where t is time in seconds. Find the midline, amplitude and period of the function f.

MAKE A GRAPH

Table 6.3

t	0	1	2	3	4	5	6	7
h	4.0	5.2	6.2	6.5	6.2	5.2	4.0	2.8
t	8	9	10	11	12	13	14	15
h	1.8	1.5	1.8	2.8	4.0	5.2	6.2	6.5

FMC 6.1B

Problems 23–26 concern a weight suspended from the ceiling by a spring. (See Figure 6.8.) Let d be the distance in centimeters from the ceiling to the weight. When the weight is motionless, $d = 10$. If the weight is disturbed, it begins to bob up and down, or *oscillate*. Then d is a periodic function of t, time in seconds, so $d = f(t)$.

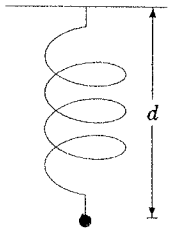


Figure 6.8

23. Determine the midline, period, amplitude, and the minimum and maximum values of f from the graph in Figure 6.9. Interpret these quantities physically; that is, use them to describe the motion of the weight.

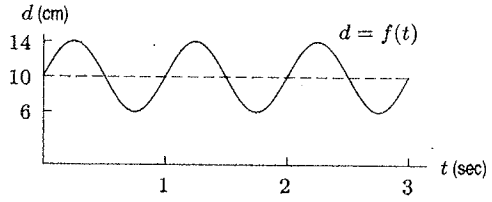


Figure 6.9

24. A new experiment with the same weight and spring is represented by Figure 6.10. Compare Figure 6.10 to Figure 6.9. How do the oscillations differ? For both figures, the weight was disturbed at time $t = -0.25$ and then left to move naturally; determine the nature of the initial disturbances.

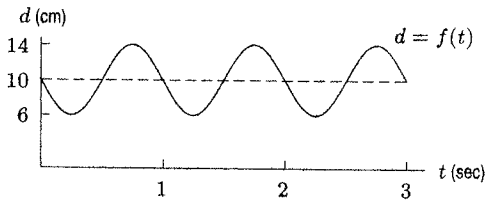


Figure 6.10

25. The weight in Problem 23 is gently pulled down to a distance of 14 cm from the ceiling and released at time $t = 0$. Sketch its motion for $0 \leq t \leq 3$.

26. Figures 6.11 and 6.12 describe the motion of two different weights, A and B, attached to two different springs. Based on these graphs, which weight:

(a) Is closest to the ceiling when not in motion?
(b) Makes the largest oscillations?
(c) Makes the fastest oscillations?

NOTE VERTICAL SCALES

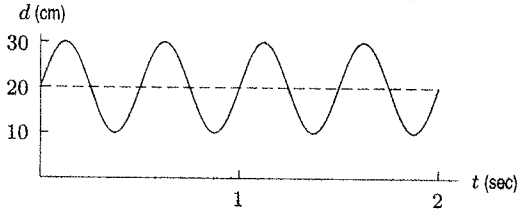


Figure 6.11: Weight A

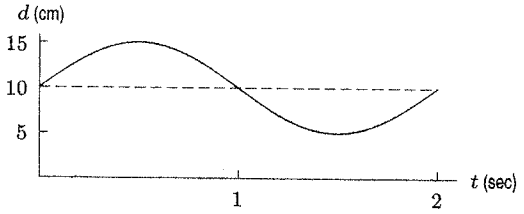
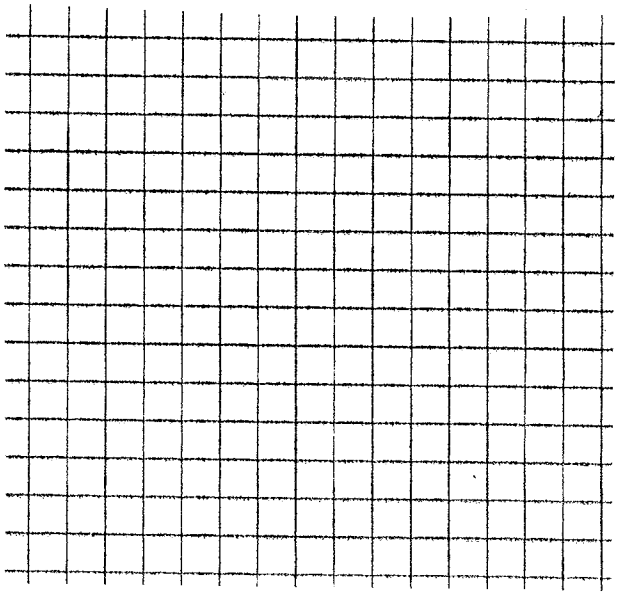


Figure 6.12: Weight B



FMC 6.2A

28. Find an angle ϕ, with $0° < \phi < 360°$, that has the same
(a) Cosine as $53°$ (b) Sine as $53°$

30. A revolving door (which rotates counterclockwise in Figure 6.28) was designed with five equally spaced panels for the entrance to the Pentagon. The arcs BC and AD have equal length.

(a) What is the angle between two adjacent panels?

(b) A four-star general enters by pushing on the panel at point B, and leaves the panel at point D. What is the angle of rotation?

(c) With the door in the position shown in Figure 6.28, an admiral leaves the Pentagon by pushing the panel between A and D to point B. What is the angle of rotation?

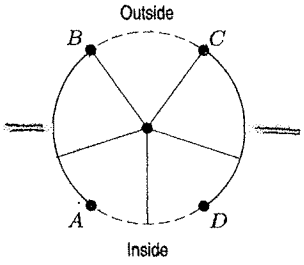


Figure 6.28

31. For the angle ϕ shown in Figure 6.29, sketch each of the following angles.

(a) $180+\phi$ **(b)** $180-\phi$ **(c)** $90-\phi$ **(d)** $360-\phi$

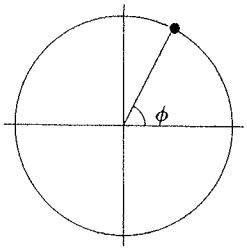


Figure 6.29

32. Let θ be an angle in the first quadrant, and suppose $\sin\theta = a$. Evaluate the following expressions in terms of a. (See Figure 6.30.)

(a) $\sin(\theta + 360°)$ **(b)** $\sin(\theta + 180°)$
(c) $\cos(90° - \theta)$ **(d)** $\sin(180° - \theta)$
(e) $\sin(360° - \theta)$ **(f)** $\cos(270° - \theta)$

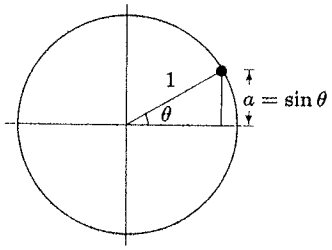


Figure 6.30

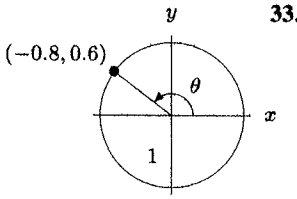
FMC 6.3A

29. Without using a calculator, give the sign of each of the following numbers:

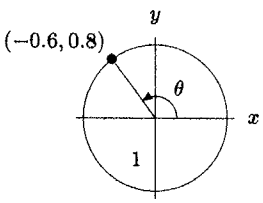
(a) $\cos 3$ **(b)** $\sin 4$ **(c)** $\sin(-4)$ **(d)** $\cos 7$

Evaluate $\sin\theta$ and $\cos\theta$ for the angle θ on the unit circle in Problems 32–33.

32.



33.



34. An ant starts at the point $(1, 0)$ on the unit circle and walks counterclockwise a distance of 3 units around the circle. Find the x and y coordinates (accurate to 2 decimal places) of the final location of the ant.

36. For ϕ in Figure 6.43, sketch the following angles.

(a) $\pi + \phi$ **(b)** $\pi - \phi$

(c) $\pi/2 - \phi$ **(d)** $2\pi - \phi$

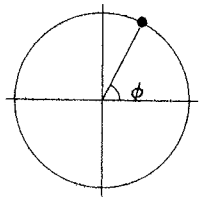


Figure 6.43

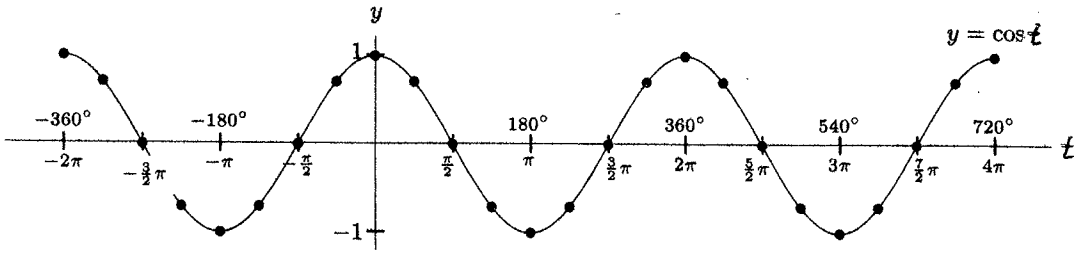
38. How far does the tip of the minute hand of a clock move in 35 minutes if the hand is 6 inches long?

44. Do you think there is a value of t for which $\cos t = t$? If so, estimate the value of t. If not, explain why not.

USE RADIANS.

44b, What about $f(t) = \frac{\sin(t)}{t}$?

- does this have a value at $t=0$?
- What about "near" $t=0$?



FMC 6.4A

27. **(a)** Match the lengths p, q, r, s marked on the unit circle in Figure 6.55 with the following values:

(i) $t = 0.8$ (ii) $t = \pi - 2.9$

(iii) $\cos(0.8)$ (iv) $-\cos(2.9)$

(b) On a graph of $y = \cos t$, sketch segments corresponding to each of the values in part (a).

graph

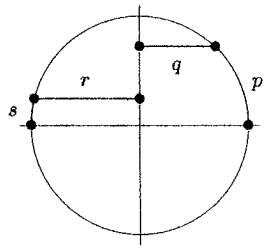


Figure 6.55

28. **(a)** Write an expression for the slope of the line segment joining P and Q in Figure 6.56.

(b) Evaluate your expression for $a = \pi/4$, $b = 4\pi/3$. Give an exact value for your answer.

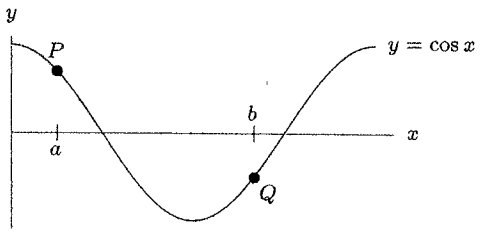


Figure 6.56

16. Match each of the letters A–G in Figure 6.49 to one of the following values of x (in radians): $1, 2, 4, 5, \pi/2, \pi$, and $3\pi/2$.

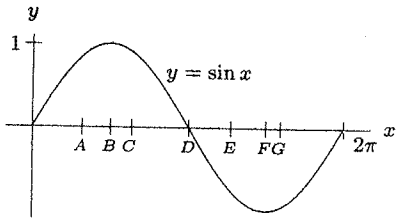


Figure 6.49

20. Figure 6.53 shows $y = \sin(x - \frac{\pi}{2})$ and $y = \sin(x + \frac{\pi}{2})$ starting at $x = 0$. Identify which is which.

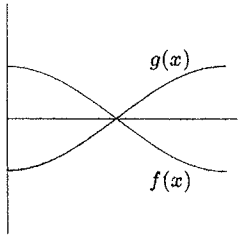


Figure 6.53

30. A circle of radius 5 is centered at the point $(-6, 7)$. Find a formula for $f(\theta)$, the x-coordinate of the point P in Figure 6.58.

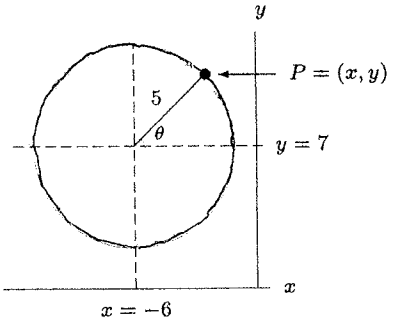


Figure 6.58

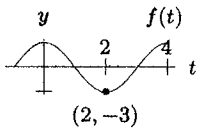
FMC 6.5A

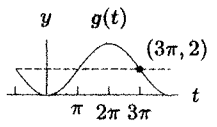
In Exercises 1–4, state the period, amplitude, and midline.

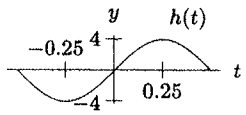
1. $y = 7\sin(4(t+7)) - 8$
2. $y = 6\sin(t+4)$
3. $y = \pi\cos(2t+4) - 1$
4. $2y = \cos(8(t-6)) + 2$

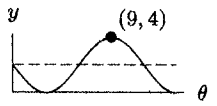
Also state the horizontal shift and phase shift. (See below ↓)

In Exercises 13–20, find formulas for the trigonometric functions.

13. 

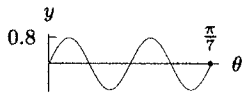
14. 

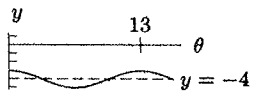
15. 

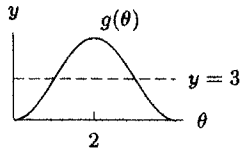
16. 

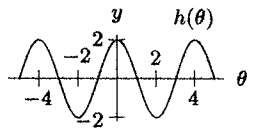
32. Find a possible formula for the trigonometric function whose values are in the following table.

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$g(x)$	2	2.6	3	3	2.6	2	1.4	1	1	1.4	2

17. 

18. 

19. 

20. 

38. The pressure, P (in lbs/ft^2), in a pipe varies over time. Five times an hour, the pressure oscillates from a low of 90 to a high of 230 and then back to a low 90. The pressure at $t = 0$ is 90.
   (a) Graph $P = f(t)$, where t is time in minutes. Label your axes.
   (b) Find a possible formula for $P = f(t)$.
   (c) By graphing $P = f(t)$ for $0 \le t \le 2$, estimate when the pressure first equals 115 lbs/ft^2.

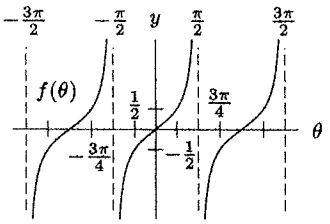
$y = A\sin(B(t-h)) + k,$
$y = A\cos(B(t-h)) + k.$

Amplitude $= |A|$; period $= 2\pi/|B|$; midline $y = k$.
Phase shift $= Bh$; horizontal shift $= h$.

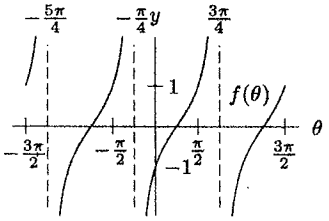
FMC 6.6A

In Exercises 16–17, give a possible formula for the function.

16.



17.



30. **(a)** Find an equation for the line l in Figure 6.76.
(b) Find the x-intercept of the line.

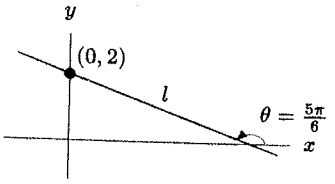
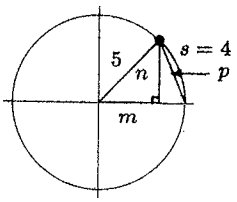


Figure 6.76

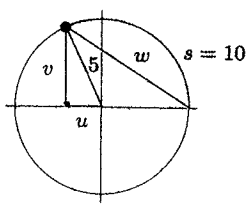
22. **(a)** $\cos\alpha = -\sqrt{3}/5$ and α is in the third quadrant. Find exact values for $\sin\alpha$ and $\tan\alpha$.
(b) $\tan\beta = 4/3$ and β is in the third quadrant. Find exact values for $\sin\beta$ and $\cos\beta$.

Find exact values for the lengths of the labeled segments in Problems 34–35.

34.



35.



Problems 26–29 give an expression for one of the three functions $\sin\theta$, $\cos\theta$, or $\tan\theta$, with θ in the first quadrant. Find expressions for the other two functions. Your answers will be algebraic expressions in terms of x.

26. $\sin\theta = x/3$

27. $\cos\theta = 4/x$

28. $x = 2\cos\theta$

29. $x = 9\tan\theta$

FMC 6.7A

36. In your own words, explain what each of the following expressions means. Evaluate each expression for $x = 0.5$. Give an exact answer if possible.

(a) $\sin^{-1} x$ **(b)** $\sin(x^{-1})$ **(c)** $(\sin x)^{-1}$.

Solve the equations in Problems 29–32 for $0 \le t \le 2\pi$. First estimate answers from a graph; then find exact answers.

29. $\cos(2t) = \frac{1}{2}$

30. $\tan t = \frac{1}{\tan t}$

31. $2 \sin t \cos t - \cos t = 0$

32. $3 \cos^2 t = \sin^2 t$

35. Approximate the x-coordinates of points P and Q shown in Figure 6.90, assuming that the curve is a sine curve. [Hint: Find a formula for the curve.]

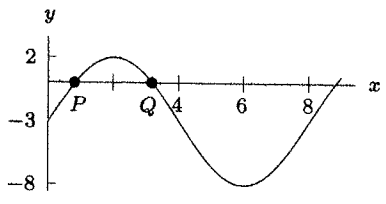


Figure 6.90

44. You are perched in the crow's nest, C, on top of the mast of a ship, S. See Figure 6.91. You will calculate how far you can see when you are x meters above the surface of the ocean.

(a) Find formulas for d, the distance you can see to the horizon, H, and l, the distance to the horizon along the earth's surface, in terms of x, the height of the ship's mast, and r, the radius of the earth.

(b) How far is the horizon from the top of a 50-meter mast? How far, measured along the earth's surface, is the horizon from the ship's position on the ocean? Use $r = 6{,}370{,}000$ meters.

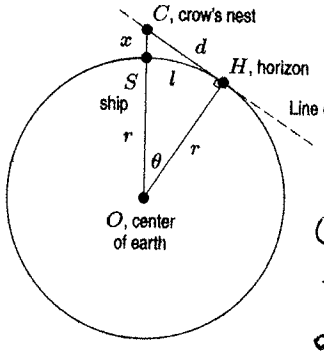


Figure 6.91

(c) Try to find a simple approximate formula

45. Let k be a positive constant and t be an angle measured in radians. Consider the equation

$$k \sin t = t^2.$$

(a) Explain why any solution to the equation must be between $-\sqrt{k}$ and $\sqrt{k}$, inclusive.

(b) Approximate every solution to the equation when $k = 2$.

(c) Explain why the equation has more solutions for larger values of k than it does for small values.

(d) Approximate the least value of k, if any, for which the equation has a negative solution.

FMC7.5A

Convert the polar coordinates in Exercises 14–17 to Cartesian coordinates. Give exact answers.

14. $(1, 2\pi/3)$ **15.** $(\sqrt{3}, -3\pi/4)$

16. $(2\sqrt{3}, -\pi/6)$ **17.** $(2, 5\pi/6)$

Convert the Cartesian coordinates in Problems 18–21 to polar coordinates.

18. $(1, 1)$ **19.** $(-1, 0)$

20. $(\sqrt{6}, -\sqrt{2})$ **21.** $(-\sqrt{3}, 1)$

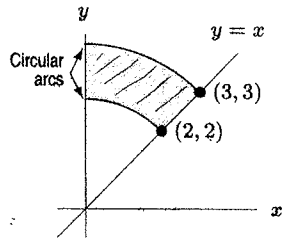
For Problems 22–28, the origin is at the center of a clock, with the positive x-axis going through 3 and the positive y-axis going through 12. The hour hand is 3 cm long and the minute hand is 4 cm long. What are the Cartesian coordinates and polar coordinates of the tips of the hour hand and minute hand, H and M, respectively, at the following times?

22. 12 noon **23.** 3 pm **24.** 9 am **25.** 1:30 pm

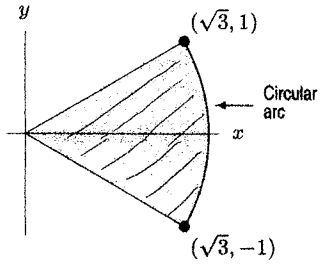
26. 7 am **27.** 3:30 pm **28.** 9:15 am

In Problems 29–31, give inequalities for r and θ which describe the following regions in polar coordinates.

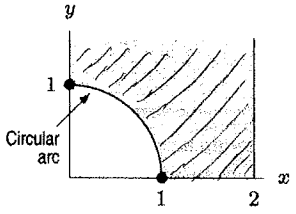
29.



30.



31.



Note: Region extends indefinitely in the y-direction.

Name: _____ Group Members: _____

Exploration 2-1a: Transformed Periodic Functions

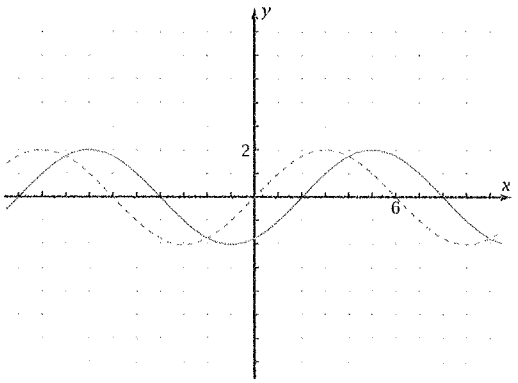
Date: _____

Objective: Given a pre-image graph and a transformed graph of a periodic function, state the transformation(s).

Give the transformation applied to $f(x)$ (dashed) to get the solid graph, $y = g(x)$.

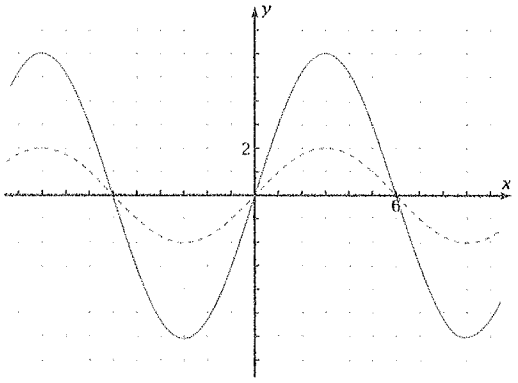
1. Verbally: _____

   Equation: $y = g(x) =$ _____



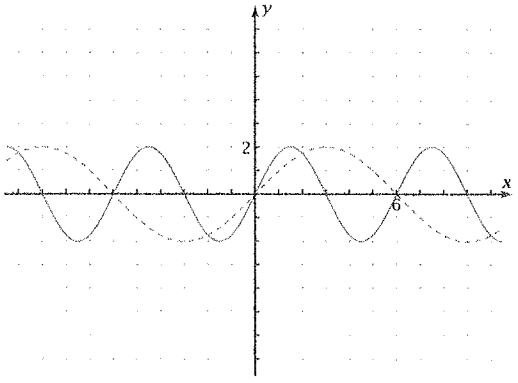
2. Verbally: _____

   Equation: $y = g(x) =$ _____



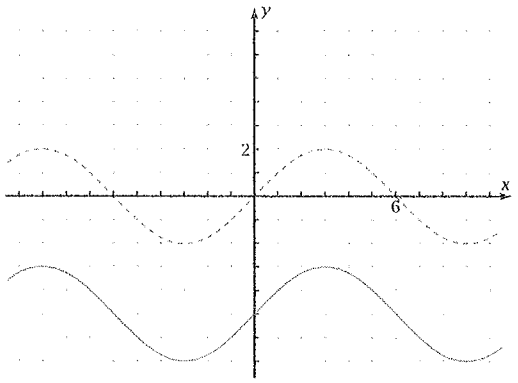
3. Verbally: _____

   Equation: $y = g(x) =$ _____



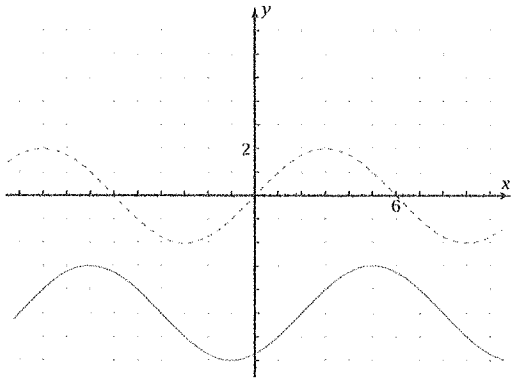
4. Verbally: _____

   Equation: $y = g(x) =$ _____



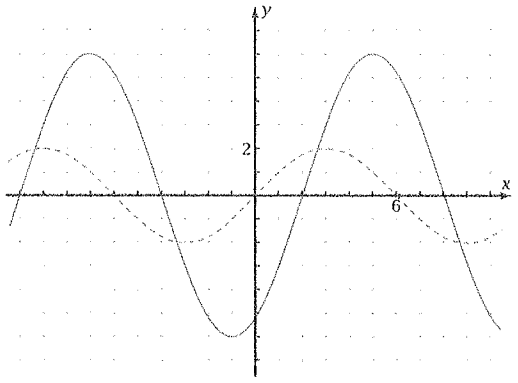
5. Verbally: _____

   Equation: $y = g(x) =$ _____



6. Verbally: _____

   Equation: $y = g(x) =$ _____



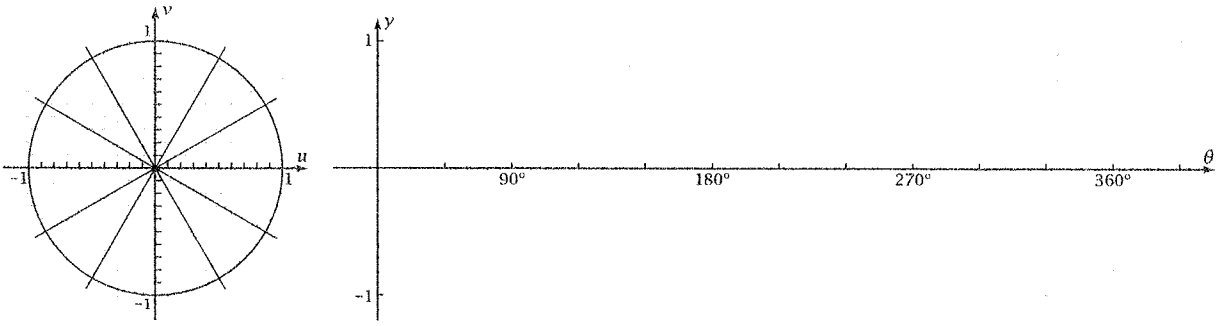
7. What did you learn as a result of doing this Exploration that you did not know before?

Name: _____ Group Members: _____

Exploration 2-3b: *uv*-Graphs and θy-Graphs of Sinusoids

Date: _____

Objective: Show a geometric relationship between angles plotted as angles and angles plotted along the θ-axis.



1. The left figure shows a unit circle in a *uv*-diagram with angles marked at every 30°. Read, to two decimal places, the coordinates (u, v) of the point where the ray at 60° cuts the unit circle.

2. Find cos 60° and sin 60° with your calculator. Explain how these numbers relate to the answers to Problem 1.

3. Plot the point $(\theta, y) = (60°, \sin 60°)$ on the θy-coordinate system on the right at the top of this Exploration. Draw a line segment showing how this point is related to the point you plotted in Problem 1.

4. Without actually calculating any more values, plot points on the graph of $y = \sin\theta$ for each 30° from 0° to 360°. Show segments connecting the appropriate points on the *uv*-diagram with points in the θy-diagram.

5. Connect the points in Problem 4 with a smooth curve. What geometrical figure is this curve?

6. Use your observation in Problem 2 to plot points on the graph of $y = \cos\theta$ for each 30° from $\theta = 0°$ to $\theta = 360°$. Connect the points with a smooth curve.

7. What transformation could you apply to the graph of $y = \sin\theta$ to get the graph of $y = \cos\theta$?

8. Explain the difference between the way the value of θ appears on the *uv*-diagram and the way it appears on the θy-diagram.

9. Why do you think the letters u and v, rather than the more common letters x and y, are used in the figure on the left at the top of this Exploration?

10. What did you learn as a result of doing this Exploration that you did not know before?

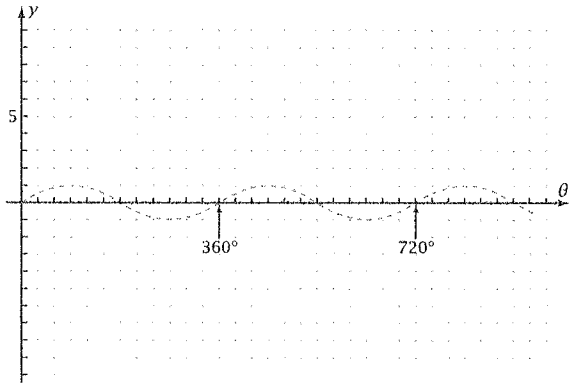
Name: _____ Group Members: _____

Exploration 2-3c: Parent Sinusoids

Date: _____

Objective: Explore the graph of the parent function $y = \sin x$, and transform the graph.

1. The graph shows the function $y = \sin x$. Plot this graph as y_1 on your grapher. Use the window shown. Turn on the grid to get the dots. Does your graph agree with this figure? _____



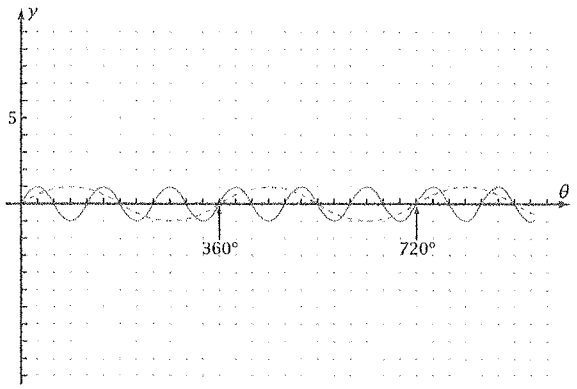
2. The **amplitude** of a periodic function is the vertical distance from the central axis to a high or low point. What is the amplitude of the sine function in Problem 1? Write the equation of the transformed function that would have an amplitude of 5.

3. Plot the transformed graph as y_2 on your grapher. Does the resulting graph really have an amplitude of 5? _____

4. The solid graph shows a transformation of the sine function from Problem 1. Identify the transformation, and write the equation for the transformed graph. Confirm that your answer is correct by plotting your equation as y_3.

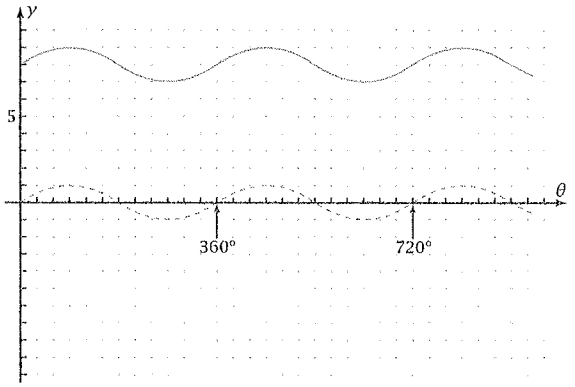
   Verbally: _____

   Equation: _____

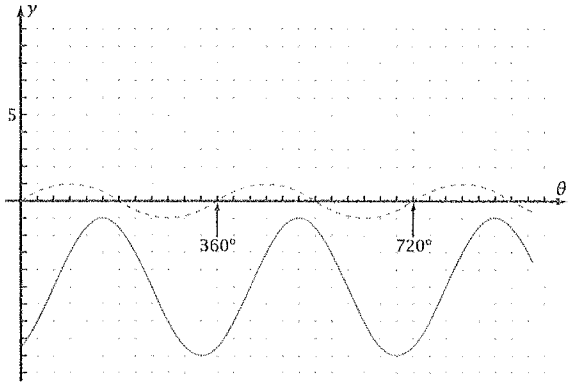


5. Write the equation for this transformed graph. Duplicate this graph on your grapher.

   Equation: _____



6. The dotted graph shows the result of three transformations. State each transformation, write the equation of the transformed graph, and duplicate the graph on your grapher.



7. Degrees can be used to measure **rotation.** What do you think is the significance of the fact that the **period** of the sine function in Problem 1 is 360°?

8. What did you learn as a result of doing this Exploration that you did not know before?

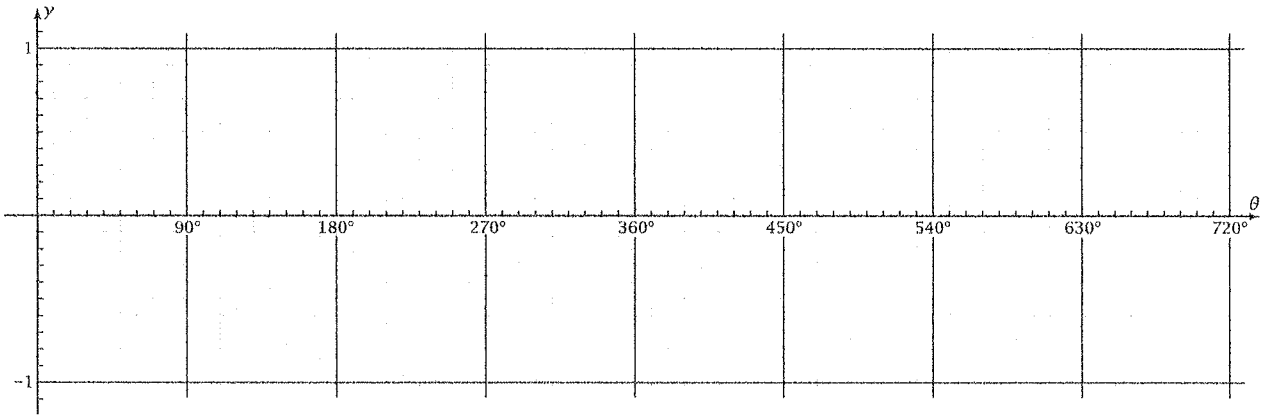
Name: _____ Group Members: _____

Exploration 3-1b: Sine and Cosine Graphs, Manually

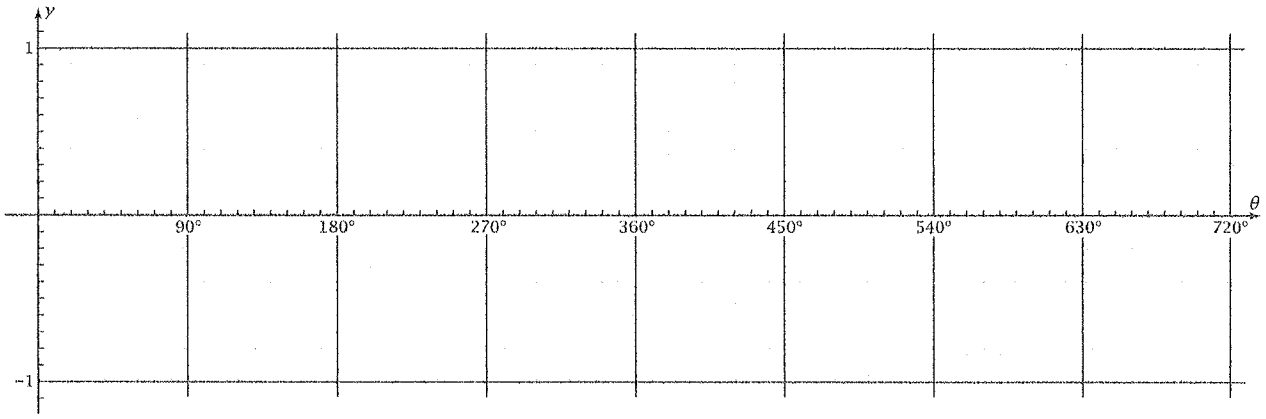
Date: _____

Objective: Find the shape of sine and cosine graphs by plotting them on graph paper.

1. On your grapher, make a table of values of $y = \sin\theta$ for each 10° from 0° to 90°. Set the mode to round to 2 decimal places. Plot the values on this graph paper. Also plot $y = \sin\theta$ for each 90° through 720°. Connect the points with a smooth curve, observing the shape you plotted for 0° to 90°.



2. Plot the graph of $y = \cos\theta$ pointwise, the way you did for sine in Problem 1.



3. Find sin 45° and cos 65°. Show that the corresponding points are on the graphs in Problems 1 and 2, respectively.

4. Find the inverse trigonometric functions $\theta = \sin^{-1} 0.4$ and $\theta = \cos^{-1} 0.8$ Show that the corresponding points are on the graphs in Problems 1 and 2, respectively.

5. What are the ranges of the sine and cosine functions?

6. Name a real-world situation where variables are related by a periodic graph like sine or cosine.

7. What did you learn as a result of doing this Exploration that you did not know before?

Name: _____ Group Members: _____

Exploration 3-3a: Tangent and Secant Graphs

Date: _____

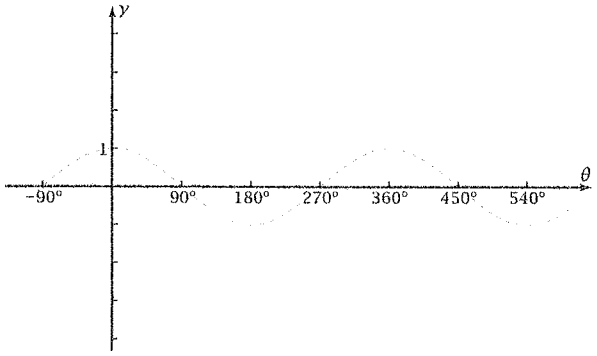
Objective: Discover what the tangent and secant function graphs look like and how they relate to sine and cosine.

No graphers allowed for Problems 1–7.

1. The reciprocal property states that

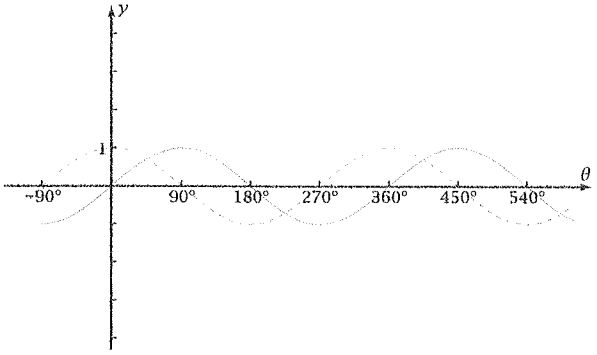
$$\sec\theta = \frac{1}{\cos\theta}$$

Without your grapher, use this property to sketch the graph of $y = \sec\theta$ on the same axes as the graph of the parent function $y = \cos\theta$. In particular, show what happens to the secant graph wherever $\cos\theta = 0$.



2. Write the quotient property expressing $\tan\theta$ as a quotient of two other trigonometric functions.

3. The next figure shows the parent functions $y = \sin\theta$ and $y = \cos\theta$. Based on the answer to Problem 2, determine where the asymptotes are for the graph of $y = \tan\theta$, and mark them on the figure.



4. Based on the quotient property, find out where the θ-intercepts are for the graph of $y = \tan\theta$. Mark these intercepts on the figure in Problem 3.

5. At $\theta = 45°$, $\sin\theta$ and $\cos\theta$ are equal. Based on this fact, what does $\tan 45°$ equal? Mark this point on the graph in Problem 3. Mark all other points where $|\sin\theta| = |\cos\theta|$.

$\tan 45° =$ _____

6. Use the points and asymptotes you have marked to sketch the graph of $y = \tan\theta$ on the figure in Problem 3. (No graphers allowed!)

7. Check your graphs with your instructor. _____

Graphers allowed for the remaining problems.

8. On your grapher, plot the graph of $y = \csc\theta$. Sketch the result here.

9. On your grapher, plot the graph of $y = \cot\theta$. Sketch the result here.

10. At what values of θ are the points of inflection for $y = \tan\theta$? Explain why the tangent function has no critical points.

11. Explain why the graph of $y = \sec\theta$ has no points of inflection, even though the graph goes from concave up to concave down at various places.

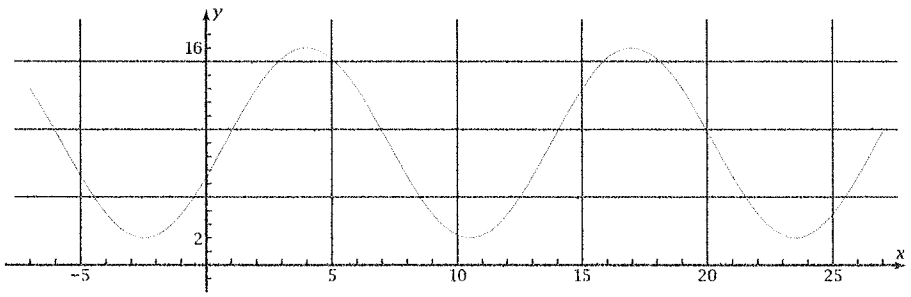
12. What did you learn as a result of doing this Exploration that you did not know before?

Name: _____ Group Members: _____

Exploration 3-6b: Given y, Find x Algebraically

Date: _____

Objective: Given the particular equation for a sinusoid and a value of y, calculate the corresponding x-values algebraically.



1. The sinusoid has equation

$$y = 9 + 7\cos\frac{2\pi}{13}(x - 4)$$

Confirm that this equation gives the correct value of y when $x = 15$.

2. Your objective is to find algebraically the values of x given $y = 5$. Substitute 5 for y. Then do the algebra necessary to get x using an arccosine. Write the **general solution** in the form

x = (number) + (period)n or (number) + (period) n

3. Write the two values of x from the general solution in the $n = 0$ row of this table. By adding and subtracting multiples of the period, fill in the other rows in the table with more possible values of x.

n	x_1	x_2
−1		
0		
1		
2		

4. Circle the points on the given graph where the line $y = 5$ cuts the graph. For each point, tell the value of n at that point.

5. Find the two values of x if $n = 100$.

6. Find the first value of x greater than 1000 for which $y = 5$. What does n equal there?

7. What did you learn as a result of doing this Exploration that you did not know before?

Name: _____ Group Members: _____

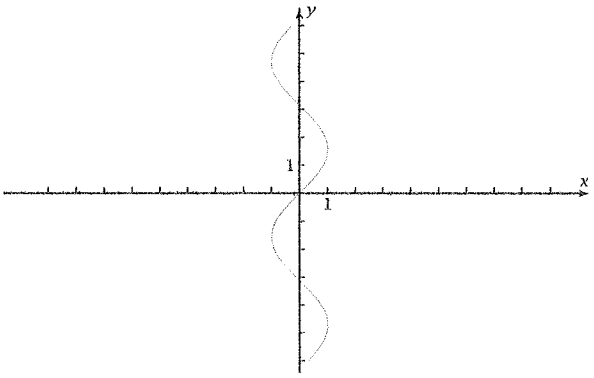
Exploration 4-6b: Principal Branches of Inverse Trigonometric Relations

Date: _____

Objective: Figure out the principal branches of each of the six inverse circular functions.

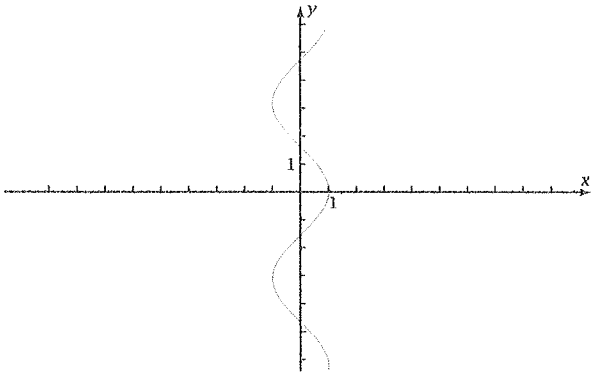
1. On your grapher, plot the inverse circular function $y = \sin^{-1} x$. Use equal scales on the two axes. On the graph of $y = \arcsin x$ shown here, darken the **principal branch** of the inverse sine relation on the part of the graph that is the inverse sine function. Give the range of the principal branch.

   Range: _____



2. Use the technique in Problem 1 to find the range of $y = \cos^{-1} x$. Darken the principal branch on the graph of $y = \arccos x$, shown next.

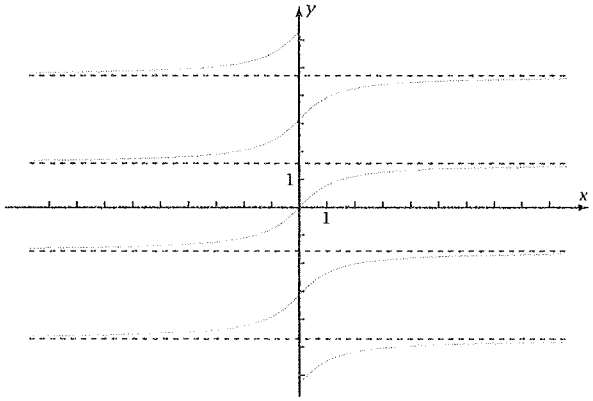
   Range: _____



3. Why can't the range of the inverse cosine function ($y = \cos^{-1} x$) be the same as the range of the inverse sine function ($y = \sin^{-1} x$)?

4. Use the technique in Problems 1 and 2 to find the range of $y = \tan^{-1} x$. Darken the principal branch on this graph of $y = \arctan x$.

   Range: _____



5. The range of $y = \sin^{-1} x$ is the closed interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Explain why the range of $y = \tan^{-1} x$ cannot include the endpoints.

6. If the range of the inverse tangent function ($y = \tan^{-1} x$) were $[0, \pi]$ (excluding $\frac{\pi}{2}$), like the range of $y = \cos^{-1} x$, then $y = \tan^{-1} x$ would still be a function. What disadvantage would there be to defining the range of $y = \tan^{-1} x$ this way?

(Over)

Name: _____ Group Members: _____

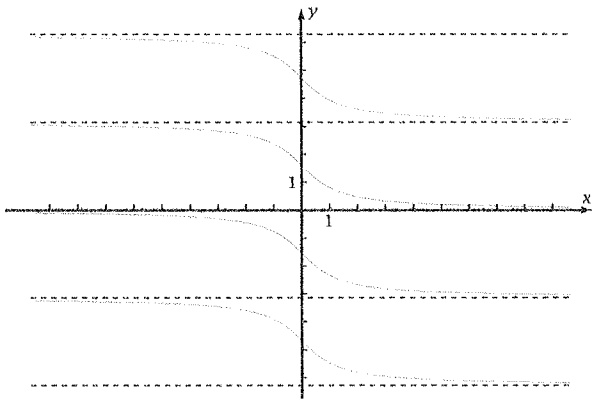
Exploration 4-6b: Principal Branches of Inverse Trigonometric Relations *continued*

Date: _____

7. Duplicate the previous graph of $y = \arctan x$ on your grapher. Give the parametric equations you used. Check your graph with your instructor. _____

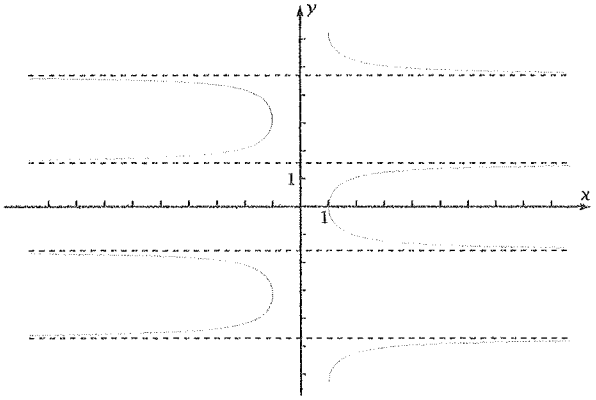
8. The graph here shows $y = \text{arccot } x$. How can you define the range of the function $y = \cot^{-1} x$ in such a way that the function is **continuous**? Darken this principal branch of $y = \text{arccot } x$.

   Range: _____



9. This next graph shows $y = \text{arcsec } x$. There is no way to restrict the range to make a continuous function $y = \sec^{-1} x$ and still use all of the domain. Darken what you think would be the best choice for the principal branch. Write the range.

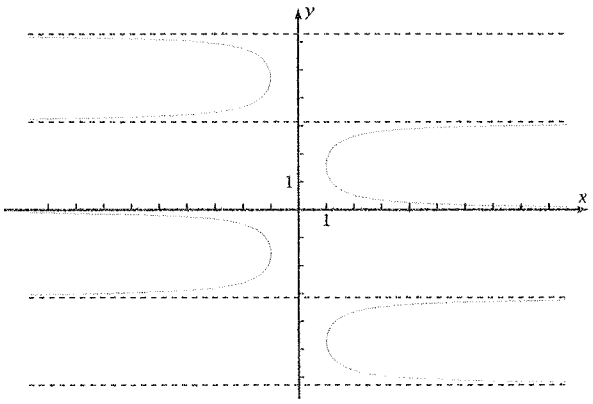
   Range: _____



10. Look in the text to find out if the principal branch you chose in Problem 9 is the commonly accepted one. _____

11. This next graph shows $y = \text{arccsc } x$. Shade what you think the principal branch is. Write the range of the function you shaded.

    Range: _____



12. Does your answer to Problem 11 agree with the range listed in the text?

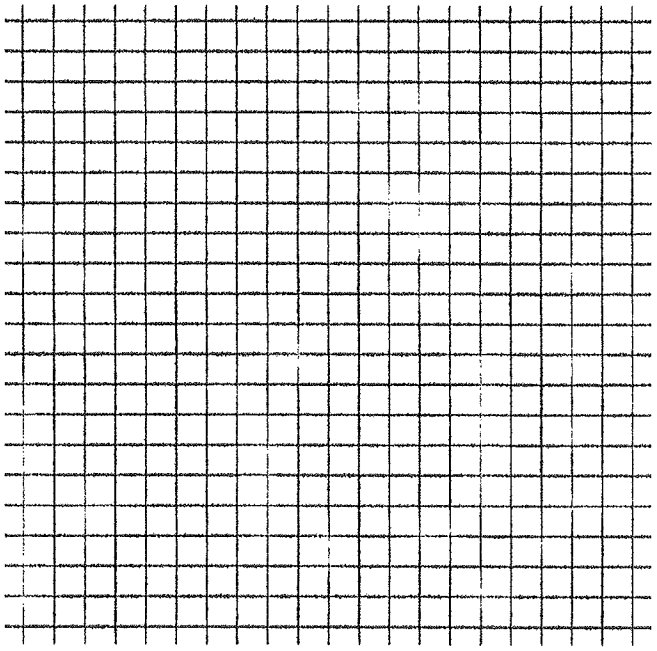
13. Look up in the text the five criteria for picking the ranges of the principal branches of the six inverse circular functions. Write the criteria here.

14. What did you learn as a result of doing this Exploration that you did not know before?

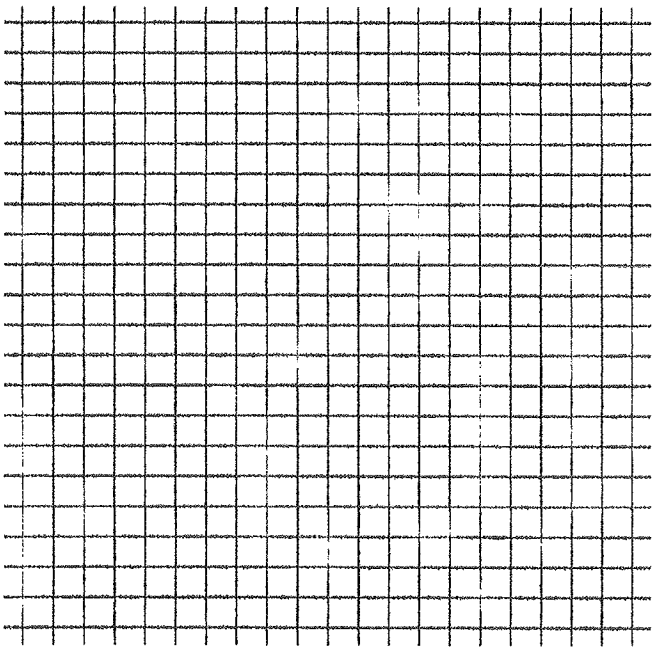
Name:_____

Sketching Angles, Circles and Arcs

1. Sketch the rays of the angle 30° in the standard position of a coordinate system.

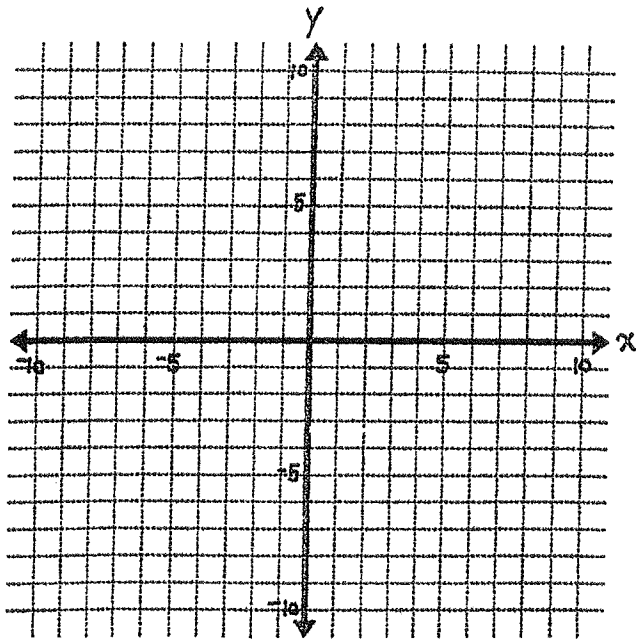


2. Sketch the angle 135° (as in question 1).
3. Sketch the angle $\frac{10\pi}{6}$ (as in question 1).



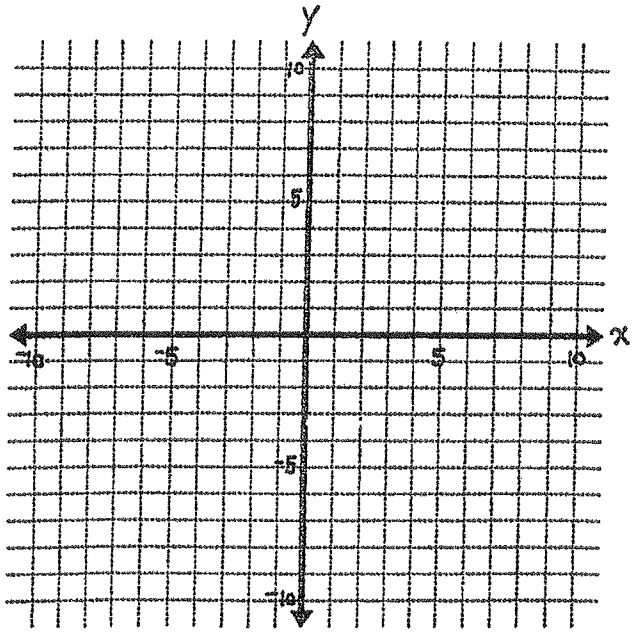
Extra sketching space above

4. Sketch a circle of radius 8 units in the coordinate system, centered at (0, 0).



Circumference = Area =

5. Sketch the sector with radius 6 units, centered at (0, 0) with central angle 60°.



Arc Length = Area =
Total Perimeter=
Values of (x, y) at top corner=

N10

ANGLE DEFINITIONS

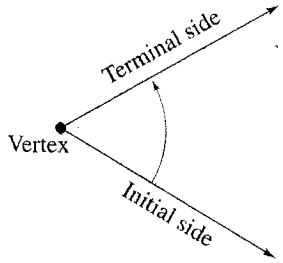


Figure 4.1

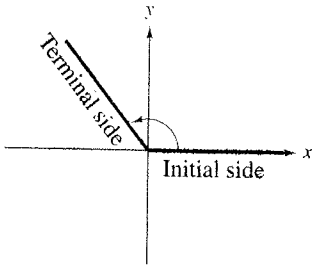


Figure 4.2

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1. The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 4.2. **Positive angles** are generated by counterclockwise rotation, and **negative angles** by clockwise rotation, as shown in Figure 4.3. Angles are labeled with Greek letters such as α (alpha), β (beta), and θ (theta), as well as uppercase letters such as A, B, and C. In Figure 4.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

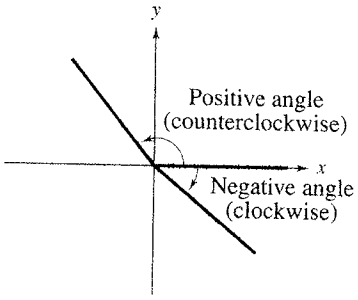


Figure 4.3

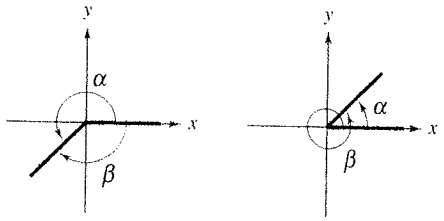


Figure 4.4

Reference Angles

The values of the trigonometric functions of angles greater than 90° (or less than 0°) can be determined from their values at corresponding acute angles called **reference angles.**

Definition of Reference Angle

Let θ be an angle in standard position. Its **reference angle** is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

Figure 4.35 shows the reference angles for θ in Quadrants II, III, and IV.

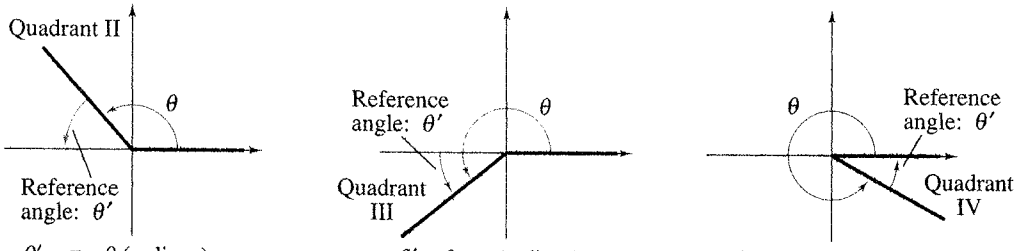


Figure 4.35

N20.

Radians and Degrees

$$1 \text{ radian (rad)} = \frac{180^\circ}{\pi}$$

radians are "unit-free" numbers.

$$1 \text{ degree (deg or } ^\circ) = \frac{\pi}{180} \text{ rad}$$

For θ_{deg} in degrees & θ_{rad} in radians:

$$\theta_{rad} = \frac{\pi}{180^\circ} \cdot \theta_{deg}$$

$$\theta_{deg} = \frac{180^\circ}{\pi} \cdot \theta_{rad}.$$

Arc Length spanned in a circle:

$s = r\theta$, where θ is in radians.

Area of a sector, given central angle and radius of a circle

$A = \frac{1}{2} r^2 \theta$, where θ is in radians.

Coterminal angles

If θ_{deg} is the degree measure of an angle, then all angles of the form

$$\theta_{deg} + 360^\circ \cdot k,$$

where k is an integer, are coterminal with θ_{deg}.

For θ_{rad} in radians, the same holds for

$$\theta_{rad} + 2k\pi.$$

NAME: _____

DIVIDING FRACTIONS AND SQUARE ROOTS

1. Rule: $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \dfrac{ad}{bc}$. Example: $\dfrac{\left(\frac{3}{5}\right)}{\left(\frac{7}{11}\right)} = \frac{3}{5}\cdot\frac{11}{7} = \frac{33}{35}$.

a) $\dfrac{\frac{4}{3}}{\frac{6}{8}} =$

b) $\dfrac{2}{\frac{11}{17}} =$

c) $\dfrac{1}{\left(\frac{1}{2}\right)} =$

d) $\dfrac{1}{\left(\frac{4}{3}\right)} =$

2. Rule: Simplify $\dfrac{1}{\sqrt{a}}$ by multiplying by $\dfrac{\sqrt{a}}{\sqrt{a}}$, $\dfrac{1}{\sqrt{a}} = \dfrac{1\cdot\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = \dfrac{\sqrt{a}}{a}$.

Example: $\dfrac{1}{\sqrt{11}} = \dfrac{1\cdot\sqrt{11}}{\sqrt{11}\cdot\sqrt{11}} = \dfrac{\sqrt{11}}{11}$.

a) $\dfrac{1}{\sqrt{2}} =$

b) $\dfrac{1}{\sqrt{3}} =$

c) $\dfrac{2}{\sqrt{3}} =$

d) $\dfrac{1}{\left(\frac{2}{\sqrt{3}}\right)} =$

e) $\sqrt{\dfrac{1}{10}} =$

f) $\sqrt{\dfrac{3}{4}} =$

g) $\sqrt{\dfrac{4}{3}} =$

h) $\dfrac{\frac{15}{17}}{\sqrt{\frac{15}{17}}} =$

N40.

Worksheet 6.3: The Unit Circle

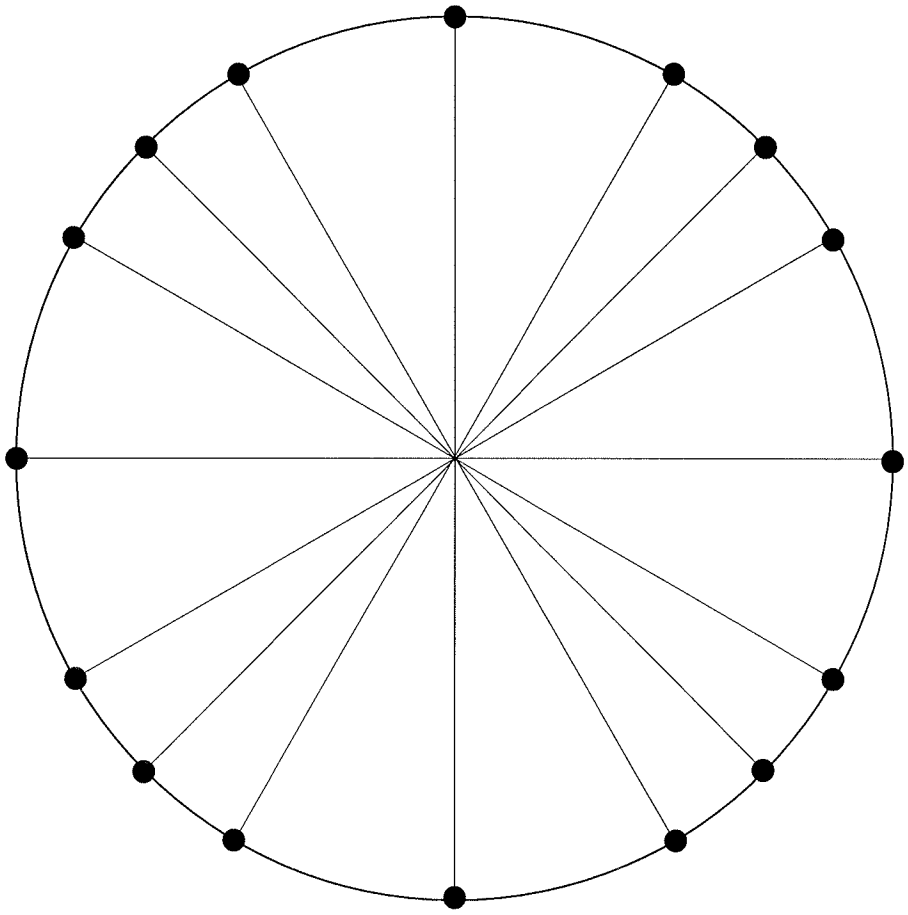


Figure 11

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\cos\theta$									
$\sin\theta$									
θ	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π	
θ	210°	225°	240°	270°	300°	315°	330°	360°	
$\cos\theta$									
$\sin\theta$									

N50.

DMS NOTATION

DEG DEGREE	MIN MINUTE	SEC SECOND
1°	$1' = \frac{1}{60}^\circ$	$1'' = \frac{1}{3600} = \frac{1}{60'}$

TI-83

Use Degree Mode (Not Radians)

MODE Degree (3rd line)

- Decimal → DMS

13.67 [2nd] [Angle] 4: ▶DMS [ENTER]

$13^\circ 40' 12''$

- DMS → Decimal

13 [ANGLE] - 1:° 40 [ANGLE] 2:' 12 [ALPHA] [+] [11] [ENTER]

13.67

degree $^\circ$ = [ANGLE] 1:°
minute ' = [ANGLE] 2:'
second '' = [ALPHA] [+] [11]

- TO FORCE DECIMAL (IN CASE STUCK IN DMS)

[MATH] [2: ▶Dec]

6.5 PRINCIPAL VALUES OF INVERSE TRIG. FUNCTIONS

6.4 Inv. trig.

PRINCIPAL VALUES.

See also:

Amc	Foerster	Finney
p. 334	p. 161	p. 161

One-to-one : useful for inverting.

ONE-TO-ONE FUNCTION	PARENT FUNCTION	RESTRICTED DOMAINS = PRINCIPAL VALUES
Sin x	sin x	$-90° \le x \le 90°$
Cos x	cos x	$0° \le x \le 180°$
Tan x	tan x	$-90° < x < 90°$
Csc x	csc x	$-90° \le x \le 90°$ and $x \neq 0°$
Sec x	sec x	$0° \le x \le 180°$ and $x \neq 90°$
Cot x	cot x	$0 < x < 180°$

COT/TAN DIFFERENCE

NOTE: CAPITAL LETTERS ONLY USED IN AMC BOOK

These are not functions : $\sin^{-1}x$, $\arcsin x$, $\cos^{-1}x$, ... etc.

These are functions : $\mathrm{Sin}^{-1}x$, $\mathrm{Arcsin}\,x$, $\mathrm{Cos}^{-1}x$, ... etc.

Calculator uses Restricted Domains.

$\mathrm{Sin}^{-1}x$: SIN^{-1}

$\mathrm{Cos}^{-1}x$: COS^{-1}

$\mathrm{Tan}^{-1}x$: TAN^{-1}

CALCULATOR CONVERSIONS

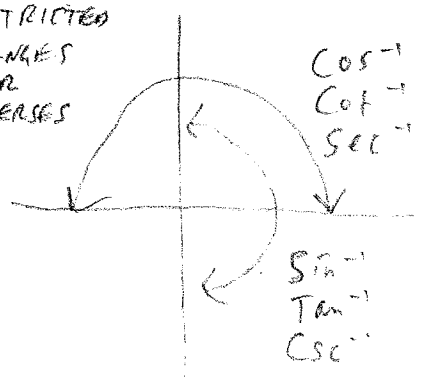
$\mathrm{Csc}^{-1}x = \mathrm{Sin}^{-1}(1/x)$

$\mathrm{Sec}^{-1}x = \mathrm{Cos}^{-1}(1/x)$

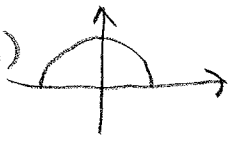
$\mathrm{Cot}^{-1}x = 90° - \mathrm{Tan}^{-1}(x) = \frac{\pi}{2} - \mathrm{Tan}^{-1}(x)$

(NOTE DIFFERENCE IN COT/TAN)

RESTRICTED RANGES FOR INVERSES



FUN PLOT : $y = \mathrm{Sin}(\mathrm{Cos}^{-1}(x))$



N70

7-4 HALF ANGLE FORMULAS p. 379.

$\cos 2\theta = 2\cos^2\theta - 1$

Let $\alpha = 2\theta$

$\cos\alpha = 2\cos^2\left(\frac{\alpha}{2}\right) - 1$

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

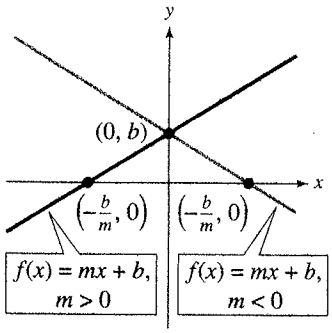
$$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad (\cos\alpha \neq -1)$$

N 80.

LIBRARY OF FUNCTIONS SUMMARY

Linear Function

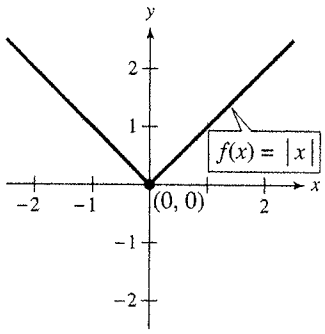
$f(x) = mx + b$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Increasing when $m > 0$
Decreasing when $m < 0$

Absolute Value Function

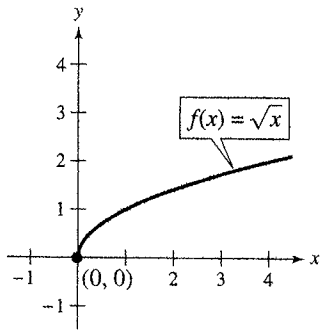
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
y-axis symmetry

Square Root Function

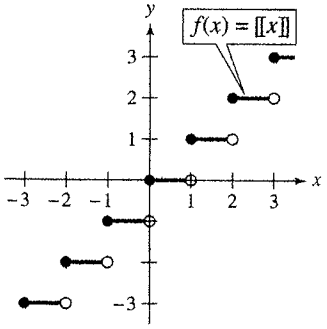
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Greatest Integer Function

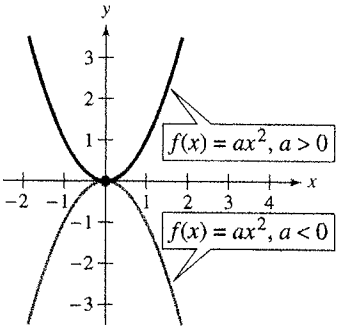
$f(x) = [\![x]\!]$



Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic (Squaring) Function

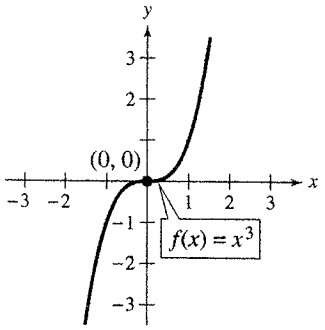
$f(x) = ax^2$



Domain: $(-\infty, \infty)$
Range ($a > 0$): $[0, \infty)$
Range ($a < 0$): $(-\infty, 0]$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$ for $a > 0$
Increasing on $(0, \infty)$ for $a > 0$
Increasing on $(-\infty, 0)$ for $a < 0$
Decreasing on $(0, \infty)$ for $a < 0$
Even function
y-axis symmetry
Relative minimum ($a > 0$), relative maximum ($a < 0$), or vertex: $(0, 0)$

Cubic Function

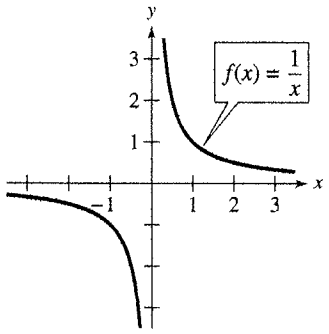
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Rational (Reciprocal) Function

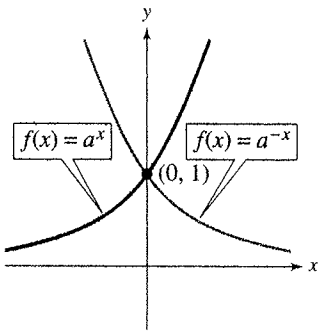
$f(x) = \frac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
No intercepts
Decreasing on $(-\infty, 0)$ and $(0, \infty)$
Odd function
Origin symmetry
Vertical asymptote: y-axis
Horizontal asymptote: x-axis

Exponential Function

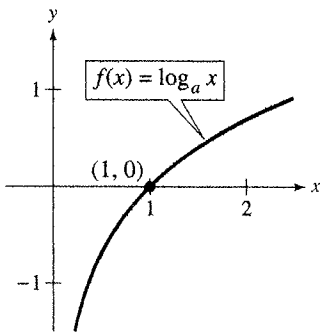
$f(x) = a^x,\ a > 0,\ a \neq 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
for $f(x) = a^x$
Decreasing on $(-\infty, \infty)$
for $f(x) = a^{-x}$
x-axis is a horizontal asymptote
Continuous

Logarithmic Function

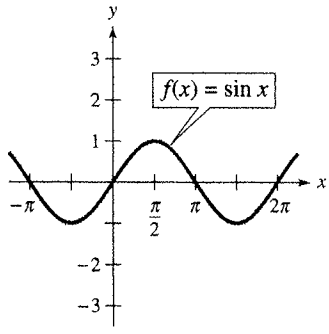
$f(x) = \log_a x,\ a > 0,\ a \neq 1$



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
y-axis is a vertical asymptote
Continuous
Reflection of graph of $f(x) = a^x$
in the line $y = x$

Sine Function

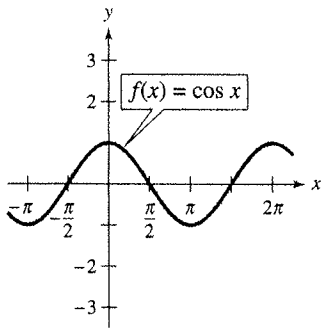
$f(x) = \sin x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Odd function
Origin symmetry

Cosine Function

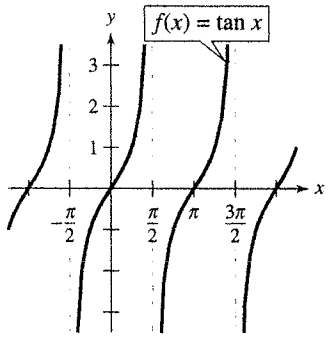
$f(x) = \cos x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$
y-intercept: $(0, 1)$
Even function
y-axis symmetry

Tangent Function

$f(x) = \tan x$

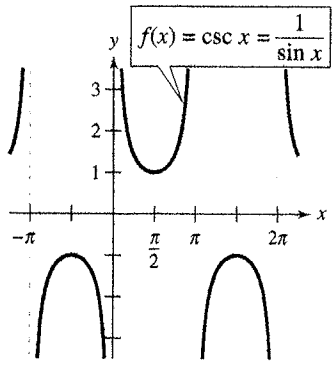


Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes:
$x = \frac{\pi}{2} + n\pi$
Odd function
Origin symmetry

N100.

Cosecant Function

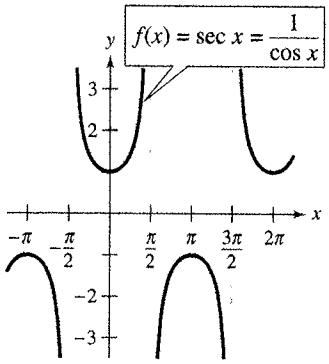
$f(x) = \csc x$



Domain: all $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
No intercepts
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Secant Function

$f(x) = \sec x$



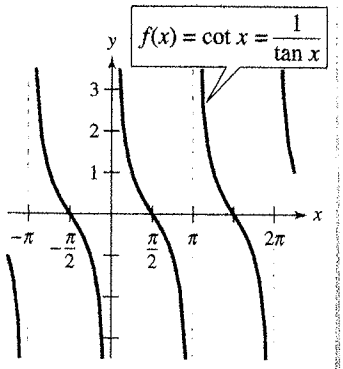
Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
y-intercept: $(0, 1)$
Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Even function
y-axis symmetry

Cotangent Function

$f(x) = \cot x$



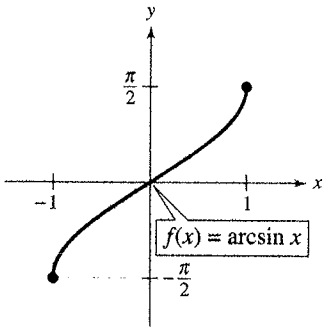
Domain: all $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π

x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$

Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Inverse Sine Function

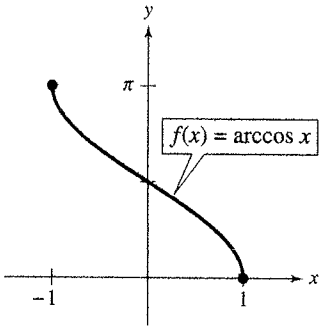
$f(x) = \arcsin x$



Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Intercept: $(0, 0)$
Odd function
Origin symmetry

Inverse Cosine Function

$f(x) = \arccos x$

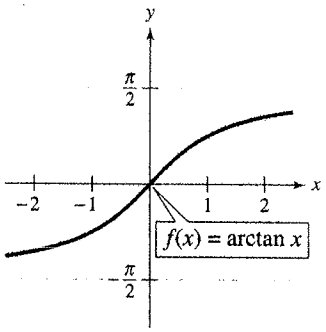


Domain: $[-1, 1]$
Range: $[0, \pi]$

y-intercept: $\left(0, \frac{\pi}{2}\right)$

Inverse Tangent Function

$f(x) = \arctan x$



Domain: $(-\infty, \infty)$
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Intercept: $(0, 0)$
Horizontal asymptotes:

$$y = \pm\frac{\pi}{2}$$

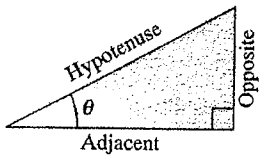
Odd function
Origin symmetry

N|0.

TRIGONOMETRY

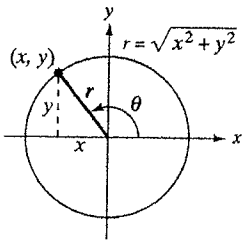
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

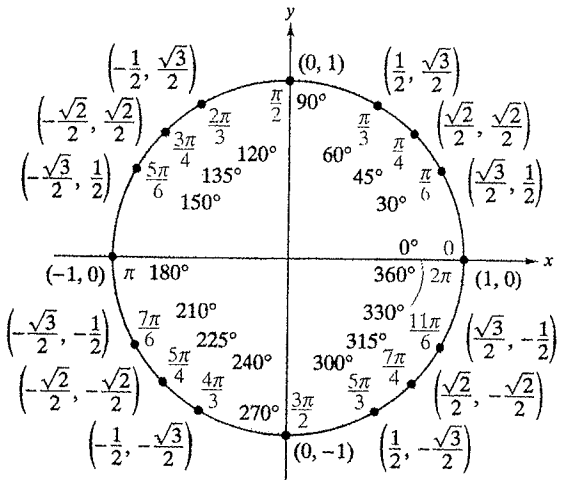


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$
$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$
$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$
$$\csc(-x) = -\csc x \quad \tan(-x) = -\tan x$$
$$\sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

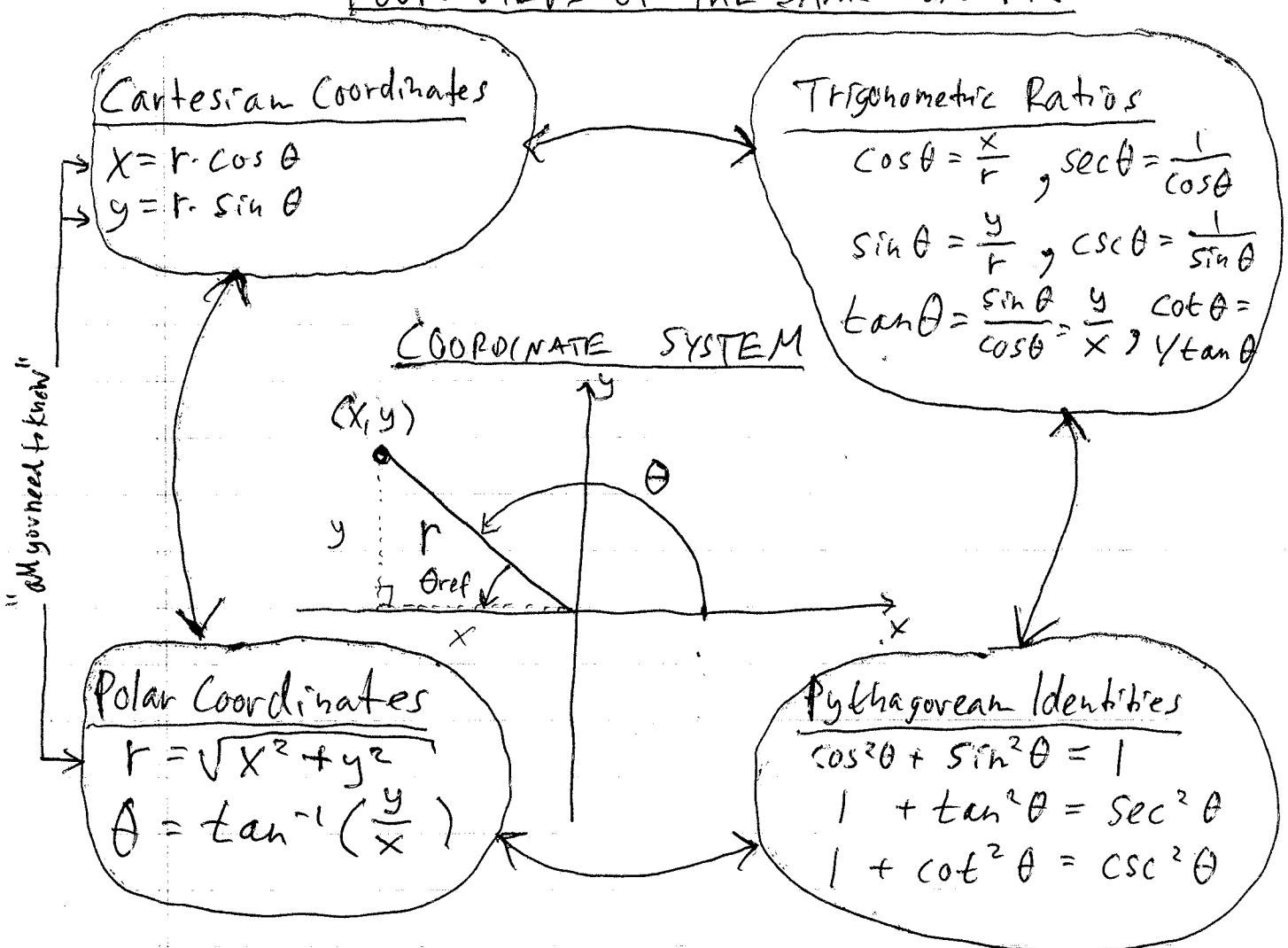
$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

N120.

FOUR VIEWS OF THE SAME CONCEPTS



FMC textbook explanations + 19 examples.

p.241 UNIT CIRCLE, $r = 1$

$x = \cos\theta$

$y = \sin\theta$

p.243 CIRCLE RADIUS r

$x = r\cos\theta$

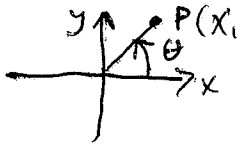
$y = r\sin\theta$

CIRCLE OF RADIUS r

p.247 RADIAN = "ANGLE IN COUNTER-CLOCKWISE DIRECTION OF A UNIT CIRCLE SPANNING AN ARC OF LENGTH 1"

p.249 Arclength, $s = r \cdot \theta_{\text{radians}}$

p.279 "Reference angle of θ is the angle between the the line joing P to the origin and the nearest part of the x-axis, $0° < \theta < 90°$ or $0 < \theta < \frac{\pi}{2}$."



p.248 $1 \text{ radian} = \frac{180°}{\pi} \approx 57.296°$

p.253 EXACT VALUES FOR SPECIAL ANGLES.

$\cos(30°) = \sqrt{3}/2$ etc.

p.267 $\tan\theta = \frac{y}{x} = \text{slope} = \frac{\sin\theta}{\cos\theta}$

p.269 $\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

N130

TRIGONOMETRIC FUNCTION GRAPHS:
CALCULATOR EXPLORATIONS WITH TI-83

RESET DEFAULT: 2nd MEM 7 2 2

- TURN ON EQUATIONS
- MODE DEGREE

2nd QUIT

MODE Degree 2nd QUIT (MODE)

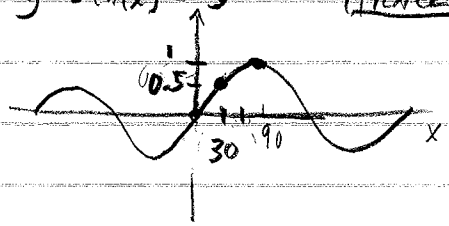
Y= $Y_1 = \sin(X)$

USE ENTER KEY TO TURN ON/OFF.

ZOOM 7:ZTRIG

TRACE

$y = \sin(x)$



TRACE ◀ ▶

WINDOW

$X_{MIN} = -352.5$
$X_{MAX} = 352.5$
$X_{SCL} = 90$

} TRY TO CHANGE THESE TO −720, 720

GRAPH

RADIANS

MODE Radian

ZOOM 7:ZTRIG

TRACE

WINDOW

$Y_{MIN} = -4$
$Y_{MAX} = 4$
$Y_{SCL} = 1$
$X_{RES} = 1$

} TRY TO CHANGE THESE TO −2, 2.

GRAPH

2nd TABLE (GRAPH)

MODE Degree

2nd TBLSET (WINDOW)

tblStart = 0

Δtbl = 15

Auto, Auto.

2nd TABLE (GRAPH)

BACK TO TBLSET = 0, 1

PARAMETRIC

MODE Par

Mode Degree

Mode Par

$X_{1T} = \cos(T)$

$Y_{1T} = \sin(T)$

WINDOW

TMIN 0

TMAX 720

TSTEP 7.5

XMIN −1.7

XMAX 1.7

XSCL 1

YMIN −1.1

YMAX 1.1

YSCL 1

TRACE

MODE FUNC

Back to MODE FUNC

$y_1 = \sin X$ ZOOM 7:ZTRIG

A	$y_2 = 2\sin X$	Vertical stretch
B	$y_2 = 0.5\sin X$	Shrink
C	$y_2 = \sin(2x)$	horizontal shrink (period shorter)
D	$y_2 = \sin(0.5x)$	stretch (period longer)
E	$y_2 = \sin(x) + 2$	vertical shift up
F	$y_2 = \sin(x) - 2$	down
G	$y_2 = \sin(X - 40)$	horizontal shift right
H	$y_2 = \sin(X + 40)$	left
I	$y_2 = 1.5\sin(0.5(X - 40)) + 2$	
J	$y_2 = \cos(x)$	
K	$y_2 = \cos(90 - X) - 2$	
L	$y_2 = \tan(x)$	

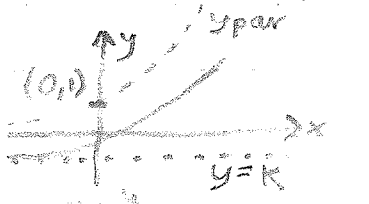
Functions: One-page Summary

8/29/07
9/28/07 V.2.

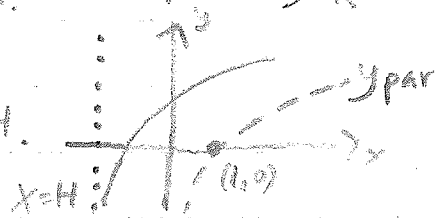
H, K = HORIZ. + VERT. SHIFTS. A = VERT. STRETCH. B = HORIZ. SHRINK

	PARENT (y_{par})	CHILD FUNCTIONS
LINE	$y = mx$	$y - K = m(x - H)$
EXP: ONE-TO-ONE	$y = e^x$	$y - K = Ae^{Bx}$
LOG-ARITHM	$y = \ln(x)$	$y - K = \ln(x - H)$
QUADRATIC	$y = x^2$	$y - K = A(x - H)^2$
RECIPROCAL	$y = \frac{1}{x}$	$y - K = \frac{1}{x - H}$
SINE	$y = \sin(x)$	$y - K = A\sin[B(x - H)]$

point: (H,K). slope: m. y-intercept: $b = K - mh$.

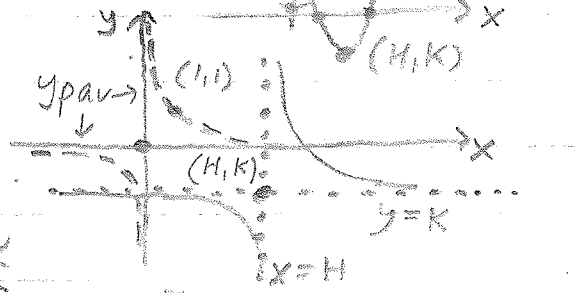


Horizontal asymptote $y = K$.
Continuous rate constant $= B$.



Vertical asymptote $x = H$.

Vertex: (H,K). A = Vertical Stretch.



Vert. asy: $x = H$
Horiz asy: $y = K$

$|A|$ = Amplitude. $B = \frac{2\pi}{T}$. T = period = Horiz Stretch.

FUNCTION. X (DOMAIN) $\to y = f(x)$ (RANGE), y UNIQUE. ZEROS = x when $f(x) = 0$.

INVERTIBLE (1-TO-1). $f(g(x)) = g(f(x)) = x$  $y = f(x) \leftrightarrow x = f^{-1}(y)$.

SYMMETRY. EVEN: $f(-x) = f(x)$  ODD: $f(-x) = -f(x)$ 

REFLECT ABOUT X-AXIS: $y = f(x)$ PARENT to $y = -f(x)$ CHILD 

EXP-LOG. $e^u \cdot e^v = e^{u+v}$. $(e^u)^v = e^{u \cdot v}$. $\ln(u \cdot v) = \ln(u) + \ln(v)$. $\ln(u^v) = v \cdot \ln(u)$.

CONTINUOUS GROWTH $y = Ae^{|B|t}$ DECAY $y = Ae^{-|B|t}$ B = Contin. rate const. $y = A \cdot (S)^t$ S = base $y = A\left(\frac{1}{2}\right)^{t/T}$ $y = A(2)^{t/T}$ T = half life or doubling time

COMPOUNDED (SEQUENCE / PIECEWISE FUNCTION WITH JUMPS): $y = A\left(1 + \frac{r}{100}\right)^t$. r = annual interest rate (%).

TRIG. CARTESIAN: $x = r\cos\theta$, $y = r\sin\theta$. POLAR: $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

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